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Soret-driven convection and separation of binary mixtures in a horizontal porous cavity submitted to cross heat fluxes

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ABSTRACT

An analytical and numerical study of Soret-driven convection in a horizontal porous layer saturated by a binary fluid and subjected to uniform cross heat fluxes is presented. The flow is driven by the combined buoyancy effect due to temperature and induced mass fraction variations through a binary water ethanol mixture. In the first part of the study, a closed-form analytical solution in the limit of a large aspect ratio of the cell ($A >> 1$) is developed. We are mainly concerned with the determination of the mass fraction gradient of the component of interest along the horizontal direction, which determines the species separation. In the second part, numerous numerical simulations are carried out in order to validate the analytical results and extend heat and mass transfer to an area not covered by the analytical study. Good agreement is found between analytical and numerical results concerning the species separation obtained for a unicellular flow. In this configuration, the Soret separation process is improved by two control parameters: the heat flux density imposed on the horizontal walls of the cell and the ratio, a, of heat flux density imposed on vertical walls to that on horizontal walls. The influence of the heat flux density ratio, a, on the transient regime (relaxation time) is also investigated numerically. We observe that an increase in the parameter a leads to a decrease in the relaxation time. Thus, for a cell heated from below without lateral heating, the onset of convection from the mechanical equilibrium state is analyzed. The linear stability analysis shows that the equilibrium solution loses its stability via a stationary bifurcation or a Hopf bifurcation depending on the separation ratio and the normalized porosity of the medium. The linear stability results are widely corroborated by direct 2D numerical simulations. The thresholds of various multicellular solutions are determined in terms of the governing parameters of the problem using nonlinear direct numerical simulations.

1. Introduction

The aim of this work is to study, analytically and numerically, the species separation of a binary fluid mixture saturating a horizontal porous cavity of large aspect-ratio. The Soret effect is taken into account. The convective motion is driven using constant cross fluxes of heat on the horizontal and vertical walls.

Thermodiffusion corresponds to the migration of the components in a gaseous mixture or aqueous solution under the temperature gradient. Thermodiffusion in fluids, or the Soret effect, was discovered by Ludwig and Soret [1]. This phenomenon implies that any fluctuation in temperature will induce a variation in the concentration of a binary mixture. The Soret effect is much larger than the Dufour effect in liquid binary mixtures [2].

The magnitude of the Soret effect is associated with the Soret coefficient, which is the ratio of the thermodiffusion coefficient to the mass diffusion coefficient. Numerous previous works have shown the importance of the Soret effect on the behavior of multicomponent systems submitted to a temperature gradient. The coupling between the heat and species molecular transports is described by the mass flux density vector:

$$\boldsymbol{J}_m = -\rho D \nabla C - \rho C(1-C)\ D_T \nabla T \qquad (1)$$

The first term of this equation comes from Fick's law and the second term describes the Soret effect (thermodiffusion). ∇C is the

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Nomenclature

a	Heat flux density ratio
A	Aspect ratio
K	Permeability of the porous medium
k	wave number
B_S	Mass fraction gradient in the x direction
B_T	Temperature gradient in the x direction
C_0'	Initial mass fraction
D	Mass diffusion Coefficient [m^2s^{-1}]
D_T	Thermodiffusion coefficient[$m^2(sK)^{-1}$]
g	Gravitational acceleration [ms^{-2}]
H	Height of the enclosure [m]
J_m	Mass flux density vector [$kg(m^2s)^{-1}$]
L	Length of the enclosure [m]
Le	Lewis number
P	Pressure [Pa]
Ra	Thermal Rayleigh number
S	Species separation
S_T	Soret parameter [K^{-1}]
t	Dimensionless time
T	Dimensionless temperature
T_0'	Reference temperature
(u,v)	Dimensionless velocity components in Ref. (x,z) directions

Greek symbols

β_C	Solutal expansion coefficient
β_T	Thermal expansion coefficient [K^{-1}]
λ_f'	Fluid thermal conductivity[$W(mK)^{-1}$]
λ'	Effective porous thermal conductivity [$W(mK)^{-1}$]
μ	Dynamic viscosity [Pas]
ν	Kinematic viscosity [m^2s^{-1}]
ρ	Density [kgm^{-3}]
ψ	Separation ratio
ε^*	Porosity of the medium
ε	Normalized porosity
α	Effective thermal diffusivity [m^2s^{-1}]
γ	Heat capacity ratio
φ	Stream function
φ_0	Intensity of the velocity field
σ	Temporal amplification of the perturbation
ω	Angular frequency

Superscript

$'$	For dimensional variables
0	Refers to a reference state
cr	Refers to a critical value

mass fraction gradient induced by the temperature gradient, ∇T, [3]. Although, the assumption $C(1-C)\approx C_0(1-C_0)$, where C_0 is the initial value of the mass fraction, is widely used, it remains valid within the limits of very small variations of the mass fraction around C_0. This assumption does not take account of the variation of D_T with respect to the mass fraction C, such as in water-ethanol systems (Kolonder et al. [4]), or the dependence on temperature, such as in water-NaCl close to 12 °C (Mojtabi et al. [5]). In the present work, D_T is assumed to be constant.

Despite its very small values of D_T, the Soret effect induces significant mass fraction variation in many natural or technological processes. The coupling of convection and thermodiffusion is called thermo-gravitational diffusion. This phenomenon was pointed out about eighty years ago with the experimental work of Clausius and Dickel [6], and by theoretical work of Furry et al. [7], which opened the way for the study of thermo-gravitational diffusion in vertical columns (TGC). In 1959, Lorenz and Emery [8] suggested introducing a porous medium in the thermogravitational column to reduce the convective motion. Thus, in a column with a large space between the two plates, filled with a porous medium, the fluid velocity is of the same order as TGC with a small gap. Thus, many works related to species separation using thermogravitational diffusion have been conducted in vertical columns. So the resulting flow intensity is controlled by the porous media permeability and an alternative means of control may be accessed by reducing the gravity component.

In 2003, Platten et al. [9] showed that the separation of species could be enhanced when the column was tilted. They concluded that separation could considerably increased by choosing an optimal inclination of the column. Charrier-Mojtabi et al. [10] showed that it was possible to perform the separation in a horizontal configuration when the separation ratio was negative or greater than a positive value leading to the onset of unicellular flow. They also showed that the Rayleigh number leading to optimum separation in a horizontal cell was greater than that the one obtained in a vertical cell (TGC). This allowed species separation in a thick cell (Elhajjar et al. [11]). Bennacer et al. [12] used a partitioning technique in order to enhance the separation within a vertical annular porous cylinder. They reported that the separation increased when the cylindrical annulus curvature increased and showed that the separation ability increased with a porous layer partitioning due to cross flow resulting from the co-rotative cells. Khouzam et al. [13] presented a new configuration leading to species separation in a horizontal rectangular cavity, heated from above or from below, in the presence of mixed convection (lid driven cavity). Their study showed that the separation could be increased for an optimal coupling between the imposed horizontal flow velocity and the flow induced by temperature gradient.

In Section 2, we present the physical problem and the mathematical formulation. An analytical model, based on the parallel-flow approximation for a cell with large aspect ratio, is proposed in Section 3. In Section 4, the maximum species separation is determined for optimal values of thermal Rayleigh number, Ra, and heat flux densities ratio, a. Most of the results were obtained for water-ethanol mixtures. The numerical method (FEM) employed to solve the governing equations is also presented. In Section 5-1, an analytical and numerical stability analysis is performed in order to obtain the critical values associated with the onset of the unicellular convection in the particular case ($a=0$), and either stationary or Hopf bifurcation is obtained. In section 5-2, direct numerical simulations are used to analyze multicellular flows obtained after the onset of convection.

2. Mathematical formulation

The system under study consists of a horizontal rectangular porous cavity with large aspect ratio $A=L/H$, where H, is the height of the cavity along the z-axis and L is its length along the x-axis (Fig. 1). The reference frame is in the middle of the cavity. The cavity is filled with a binary fluid mixture of density ρ and dynamic