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# Entrance effect on the interfacial heat transfer and the thermal dispersion in laminar flows through porous media

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ABSTRACT

Macroscopic coefficients that are needed to complete porous media models, such as the thermal dispersion and the interfacial heat transfer, are in general calculated under thermally and hydrodynamically fully developed conditions. In this study, a laminar flow that thermally develops in a porous structure is simulated to analyze an aspect that has not been addressed in the literature, the entrance effect on the calculation of macroscopic coefficients. Specifically, the simulation of a microscopic steady laminar flow in a porous medium formed by staggered square cylinders with $Re_D = 1$, 10 and 75, Péclet numbers in the 10-$10^4$ range, and porosities between 55 and 95% is presented. The domain simulated has been chosen large enough to allow the flow to thermally develop for large Péclet numbers. First, numerical solutions are space averaged to show that the application of the cellular average is preferred over the generally used volume average. Employing the cellular average, the interfacial heat transfer and the streamwise thermal dispersion are computed in the entire computational domain, from the inlet, where the flow is thermally developing, to the outlet, where fully developed conditions are achieved. Numerical computations for the interfacial heat transfer show a peak at the entrance that gradually decreases to a fully developed value. The value of the peak and the length of the developing region increases with the porosity and Péclet number. Therefore, it is concluded that for laminar flows and large Péclet numbers (>500), the assumption that the interfacial heat transfer is a constant defined by its fully developed value implies large errors in the calculation of the energy transferred between phases of the porous medium. The computation of the streamwise thermal dispersion shows the same developing region than that calculated for the interfacial heat transfer. In this region, this coefficient rises monotonically from zero to a fully developed value. Results show that the macroscopic modeling of laminar flows in ordered media cannot neglect the entrance effect for high porosities and large Péclet numbers.



## 1. Introduction

The analysis of heat transfer in a heterogeneous system formed by a solid matrix filled with a fluid is required in a vast number of applications of different fields (e.g. bed reactors, pebble nuclear reactors, heat exchangers, vapor generators, oil production, etc.). Generally, this complex solid matrix is difficult to simulate with precise geometrical details, or the computational effort required to do so is unreachable. These reasons have motivated the development of continuum or porous media models that represent a region of the matrix formed by several pores as a macroscopic homogeneous system with uniform properties [1]. One of the techniques to rigorously derive continuum models for multiphase systems is the volume-averaging [2]. This technique spatially smooths equations that are valid in one phase to produce equations that are valid everywhere. With this smoothing, the complexity of the geometry is avoided, but the complex physics that take place at the pore-scale still need to be represented. Therefore, partial differential equations that define porous media models, incorporate terms that represent the pore scale physics at a macroscopic level. These additional terms involve the use of macroscopic coefficients that can be calculated from theoretical analyses (e.g. Ref. [3]), experimental results (e.g. Ref. [4]) and/or numerical experiments (e.g. Ref. [5]).

The use of the volume-averaging technique to develop macroscopic equations is well documented [2]. When transport equations are volume averaged, several length-scale constrains need to be imposed to avoid a non-local problem. Quintard and Whitaker [6,7]

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**Nomenclature**

| | |
|---|---|
| $a_{sf}$ | interfacial area per unit volume |
| $h_{sf}$ | interfacial or macroscopic heat transfer coefficient |
| $k_f$ | fluid thermal conductivity |
| $k_{D\text{-}xx}$ | dispersion coefficient in the streamwise direction |
| $r_0$ | radius of averaging volume |
| $A_{sf}$ | interfacial area |
| $C_p$ | fluid specific heat |
| $CA$ | cellular average |
| $D$ | square-edge length |
| $H$ | REV's dimension (REV volume = $2H$ x $H$) |
| $P$ | pore length scale |
| $Nu_D$ | macroscopic Nusselt number ( $h_{sf} D/ k_f$ ) |
| $Nu_{D\text{-}FD}$ | fully developed Nusselt number |
| $Nu_{D\text{-}peak}$ | peak Nusselt number |
| $Pe$ | Péclet number |
| $Pe_D$ | Péclet number based on the Darcy velocity and D |
| $Re$ | Reynolds number |
| $Re_D$ | Reynolds number based on the Darcy velocity and D |
| $T_B$ | bulk temperature |
| $T_i$ | inlet fluid temperature |
| $T_w$ | wall temperature |
| $\langle T\rangle^f$ | $V_f$ -normalized space averaged temperature |
| $U$ | cross-section averaged streamwise velocity |
| $U_D$ | $V$-normalized space averaged streamwise velocity, Darcy velocity |
| $\langle U\rangle^f$ | $V_f$ -normalized space averaged streamwise velocity |
| $V$ | volume of the REV |
| $V_f$ | fluid volume inside the REV |
| $V_s$ | Solid volume inside the REV |
| $VA$ | volume average |

*Greek symbols*

| | |
|---|---|
| $\phi$ | porosity |
| $\gamma_f$ | distribution function |
| $\nu$ | kinematic fluid viscosity |
| $\theta$ | macroscopic non dimensional temperature |
| $\rho$ | fluid density |
| $\xi$ | local coordinate |

*Additional notations*

| | |
|---|---|
| $\langle\psi\rangle$ | volume average of $\psi$ |
| $\langle\psi\rangle^f$ | fluid volume average of $\psi$ |
| ${}^i\psi$ | space fluctuation of $\psi$ |
| $\overline{\psi}^A$ | space average of $\psi$ (volume average or cellular average) |

derived such constrains for ordered and disordered porous media. Additionally, they showed that the weighting function defined as cellular average (CA) is a superior tool than the volume average (VA) to perform the space averaging in ordered media. While VA variables show pore-scale fluctuations, CA variables do not. For instance, the VA value of the interfacial heat transfer coefficient is dependent on the location of the representative elementary volume (REV) in the porous structure [8]. The recommendation given by Quintard and Whitaker to employ the CA over the VA in ordered porous media has not been followed by the research community. As it is shown in the following paragraphs, there is a vast literature that employs the VA as the space average tool to derive macroscopic equations or to calculate macroscopic coefficients. In this study, the superiority of the CA is shown by comparing the VA and the CA tools in different numerical experiments, emphasizing the importance of its use.

The study of heat transfer in a saturated fluid flowing through obstacles has been frequently employed by the research community to investigate the performance of porous media models (e.g. Refs. [9,10]). When a space-averaging tool, such as VA or CA, is applied to the fluid energy equation, a macroscopic energy equation is obtained. The latter is characterized by terms that account for the interaction between phases at a pore-scale. In the general case, these terms require the definitions of three additional macroscopic coefficients to complete the macroscopic description [1]: the tortuosity, the thermal dispersion tensor and the interfacial heat transfer. These macroscopic coefficients are needed to complete macroscopic models that allow simulating the energy transfer process without modeling the pore-length-scale [11–13]. In particular, the coupling between the solid and fluid-phase temperatures is partially described with the interfacial heat transfer coefficient. Therefore, extensive literature is dedicated to the numerical computation of the interfacial heat transfer employing the VA tool. Kuwahara et al. [14] carried out numerical simulations in a periodic REV formed by two dimensional staggered squares to compute the interfacial heat transfer coefficient for a variety of flow conditions. These data allowed deriving a correlation for this coefficient dependent on the Reynolds number, Prandtl number and porosity (this correlation was later analyzed in Refs. [15,16]). The methodology employed by Kuwahara et al., which employs a single cell with periodic boundary conditions for ordered media, is attractive due to its low computational cost. This methodology has been employed frequently for different porous structures, for laminar and turbulent flows and for different flow orientations respect to the porous structure. For instance, Saito and de Lemos [17] developed a correlation for laminar and turbulent flow for the porous structure employed in Ref. [14]. Gamrat et al. [18] also investigated the heat transfer process in a bank of aligned and staggered square rods but including the effect of a volumetric heat source. The effect of the flow direction in the porous structure was considered by Alshare et al. [19]. Based on single cell numerical experiments for an aligned bank of squared rods, the interfacial heat transfer was calculated for aligned and non-aligned flows. A stronger dependence on the Reynolds numbers for non-aligned flows than for aligned flows was obtained for the coefficient. Some studies simulated several REVs of ordered media to study aspects of the heat transfer process that cannot be properly analyzed with single REV simulations. Takemoto et al. [20] simulated the flow transition between steady laminar and oscillatory laminar flow in a bank of aligned cylinders. This study revealed that the solution depends on the initial condition or on the path through which the current state was obtained. Pathak and Ghiaasiaan [21] simulated square rods in a row to study the effect of pulsating laminar flow in the computation of macroscopic parameters. Imani et al. [22] studied the effect of porosity, solid-thermal conductivity ratio and Reynolds number on the heat transferred between phases when no explicit boundary condition, such as constant temperature or constant heat flux, is imposed on the solid–fluid interface. As exemplified, calculation of the interfacial heat transfer has been analyzed abundantly for the last two decades. However, to the knowledge of the author, there are no numerical studies that quantify the entrance effect on the calculation of the interfacial heat transfer. This effect needs to be considered to understand its impact on the proper modeling of the energy transfer process in laminar flows in porous media.

Other parameter that is relevant in the study of heat transfer in porous media is the thermal dispersion tensor. For ordered media, Koch et al. [3] calculated, in the limit of high porosities and for Stokes flows, a dependence on $Pe^2$ for the streamwise component