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Boundary layer heat and mass transfer with Cattaneo–Christov double-diffusion in upper-convected Maxwell nanofluid past a stretching sheet with slip velocity

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ABSTRACT

A systematic study is presented for the boundary layer Cattaneo–Christov double-diffusion model of heat and mass transfer in an upper-convected Maxwell nanofluid over a stretching sheet. The innovative constitutive model, namely Cattaneo–Christov upper-convected material derivative, is first introduced in characterizing the boundary layer slip shear flow, thermal diffusion and nanoparticles concentration diffusion. Viscoelastic relaxation framework system of upper-convected Maxwell nanofluid is constructed uniquely in which heat and mass transfer are both determined by Cattaneo–Christov model uniformly. The effects of Brownian motion & thermophoresis and the dynamic viscosity assumed as the linear function of temperature are taken into account. The highly coupled boundary layer governing equations including momentum, energy & mass conservation equations are transformed to similarity equations via appropriate dimensionless variables. The velocity, temperature & concentration distributions affected by respective relaxation parameters are obtained by homotopy analysis method (HAM) and analyzed thoroughly. Results indicated that this viscoelastic relaxation framework system makes us possible to predict the relaxation times transport characteristics. The internal elastic stress aggregates initially (with big skin friction) and then release (small stress) along with the development of the boundary layer, which is the main reason to generate macroscopic relaxation phenomena. Moreover, the effects of velocity slip on boundary layer transport are also discussed.

1. Introduction

With the rapid progress of modern engineering technology, the nano-materials as a kind of new materials has got extensive attention due to its various applications in industrial, transportation, electronics and biomedicine. Nanofluid is one kind of nano-materials which belongs to a mixture of nanoparticles (average sizes 1–100 nm) and base fluid. Generally, the nanofluids have the higher heat conductivity efficiency than pure fluid due to a volume fraction (usually <5%) of metal nanoparticles. Heat and mass transfer of Newtonian fluid base nanofluid, such as water base nanofluid have been widely investigated. In 2009, Wong and Omar De Leon released the applications of nanofluids [1], their studies shown specific characteristics that the heat transfer rate can be enhanced by adding nanoparticles to the base fluid. Khanafer and Vafai made the detailed experimental and theoretical researches for the thermophysical properties of nanofluids [2]. The comprehensive discussion about convective transport in nanofluids especially for the Brownian diffusion, thermophoresis and diffusiophoresis were made Buongiorno [3]. The integrated study of thermophoretic mobility and interfacial nanolayer was proposed by Fu et al. and the thermophoretic velocity of nanoparticles were obtained by solving Navier–Stokes equations with velocity slip conditions [4]. Makinde et al. studied the boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, the effects of Brownian motion and thermophoresis were also analyzed [5]. Rana et al. investigated the boundary layer flow over non-linear stretching flat with effects of Brownian motion and thermophoresis [6]. The numerical analysis about convective heat transfer in a nanofluid flow over a stretching surface was carried out by Vajravelu & Pop et al. in which the velocity and temperature boundary layer thickness are dependent on the

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volume fraction of nanoparticles [7].

The homotopy analysis method (HAM) as an approximately analytical approach was employed by Alsaedi et al. to study the stagnation point flow of nanofluid near a permeable stretched surface with convective boundary condition [8]. Malvandi et al. made an analytical investigation on boundary layer flow and heat transfer of nanofluid induced by a nonlinearly stretching sheet, the effects of Brownian motion and thermophoresis are taken into account [9]. The boundary layer flow and heat transfer of a nanofluid over a stretching sheet with the velocity slip was studied numerically by Sharma, Ishak & Pop [10]. Rahmana et al. and Khairy Zaimi [11,12] investigated numerically the boundary layer flow and heat transfer of nanofluid past nonlinearly permeable shrinking/stretching sheet, the dual solutions were found to exist in a certain range of the stretching/shrinking parameter. The boundary layer flow and heat transfer of the second grade Non-Newtonian based nanofluid over the stretching sheet was studied by Mustafa et al. and Bhargava et al. by using the HAM (Homotopy Analysis Method) and VFEM (Variational Finite Element Method), respectively [13,14].

The upper-convected Maxwell fluid as one kind of viscoelastic fluid has been studied. In 2005, Sadeghy et al. [15] studied the Sakiadis flow of an upper-convected Maxwell fluid over a rigid steady moving plate. The rotating flow of the upper-convected Maxwell fluid was investigated by Sajid et al. [16]. Hayat et al. considered the MHD flow and mass transfer of an upper-convected Maxwell fluid past a porous shrinking sheet with chemical reaction [17,18]. Motsa et al. [19] considered the MHD boundary layer flow of an incompressible upper-convected Maxwell (UCM) fluid over a porous stretching surface and analyzed the effects of relaxation time by using successive Taylor series linearization method. Ramesh et al. studied numerically the influence of heat source/sink on Maxwell based nanofluid over a stretching surface, the Brownian motion and thermophoresis were taken into account for thermal diffusion [20]. The MHD boundary layer flow of Maxwell based nanofluid past a stretching sheet was studied numerically by Nadeem et al. [21].

In summary for previous study above, the researchers employ the classical heat and mass transfer models instead of considering general anomalous thermal and mass diffusion. Obviously, this is inconsistent with the fact that the changing of combination of relaxation times for velocity fields should affect both the temperature and concentration fields. Cattaneo [22] proposed a modified Fourier's law by adding a thermal relaxation characteristic time term to represent the “thermal inertia”, which is known as Maxwell–Cattaneo (MC) law. Christov replaced the partial time derivative in the M–C law with the upper-convected Oldroyd derivative, which is a frame indifferent objective rate and the frame-indifferent generalization of Fourier's law was employed in general energy equation [23]. In 2010, Straughan [24] studied the problem of thermal convection in a horizontal layer of incompressible Newtonian fluid introducing the Cattaneo–Christov heat flux theory and obtaining the numerical solutions by the D2 Chebyshev tau numerical method. Tibullo et al. figured out a uniqueness result for heat conduction of the incompressible fluids with employing Cattaneo–Christov model and some unique performance of the result was proved in detail [25]. Haddad considered the thermal instability in a Brinkman porous media using the same heat flux model [26].

In this paper, we consider the transport of an upper-convected Maxwell nanofluid over a stretching sheet with velocity slip boundary. The Cattaneo–Christov upper-convected material derivative is introduced in characterizing the constitutive relationship of the thermal diffusion and nanoparticles concentration diffusion. The effects of Brownian motion, thermophoresis and the velocity slip boundary are taken into account. In addition, the dynamic viscosity is assumed as the linear function of temperature [27,28]. The approximately analytical solutions of present problem are obtained by using HAM [29–31], which are better convergence and more theoretical significance than numerical results. The effects of pertinent parameters on the velocity, temperature fields and nanoparticles concentration in boundary layer are discussed in detail.

2. Basic governing equations

Consider the two-dimensional steady incompressible upper-convected Maxwell nanofluid past a stretching sheet with uniform surface temperature T_w. The temperature T and the volume fraction of nanoparticle C of the surface and ambient are taken as constant values T_w and T_∞, C_w and C_∞ respectively. The effects of Brownian force, thermophoretic force and the velocity slip between the nanofluid and the stretching sheet are taken into account. We keep with the assumptions that the nanoparticle concentration is dilute and the dynamic viscosity of nanofluid is a linear function of temperature only. We choose the Cartesian coordinates x-axis and y-axis along the stretching sheet and normal to it respectively and the detail schematic is depicted in Fig. 1. Then in view of the boundary layer approximations, the conservation equations of mass and momentum can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_M\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=\frac{1}{\rho_f}\frac{\partial}{\partial y}\left(\mu_{nf}\frac{\partial u}{\partial y}\right) \tag{2}$$

The stretching and slip boundary conditions are:

$$u=sx+\lambda_0\frac{2-\sigma_v}{\sigma_v}\frac{\partial u}{\partial y}\bigg|_{y=0},\quad v=0 \text{ at } y=0 \text{ and } u\to 0 \text{ as } y\to\infty \tag{3}$$

where u and v are the velocity along the x-axis and y-axis, ρ_f is the density of the base Maxwell fluid, λ_M is the relaxation time of velocity, s is a positive dimensional constant, σ_v is the tangential momentum accommodation coefficient, λ_0 is the molecular mean free path. It is assumed that the dynamic viscosity of nanofluid μ_{nf} can be simplified as the temperature-dependent form $\mu_{nf}=\mu_{f\infty}[\alpha+\beta(T_w-T)]$ and $\mu_{f\infty}$ is the constant value of dynamic viscosity far away from the sheet and $\alpha, \beta>0$ are constants [27,28].

The upper-convected material derivative of a vector can be written as [23].

$$\frac{D\mathbf{A}}{Dt}=\frac{\partial \mathbf{A}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{A}-\mathbf{A}\cdot\nabla\mathbf{V}+(\nabla\cdot\mathbf{V})\mathbf{A} \tag{4}$$

where $\mathbf{V}$ represents the velocity vector and $\mathbf{A}$ is a vector which can be replaced by the heat flux vector or the mass flux vector. The Cattaneo–Christov diffusion model is introduced in characterizing the thermal diffusion and concentration diffusion with relaxation of heat flux and mass flux respectively. Then the frame-indifferent generalization of Fourier's law and Fick's law, namely Cattaneo–-Christov anomalous diffusion model, are derived as

$$\mathbf{q}+\lambda_E\left[\frac{\partial \mathbf{q}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{q}-\mathbf{q}\cdot\nabla\mathbf{V}+(\nabla\cdot\mathbf{V})\mathbf{q}\right]=-k_f\nabla T \tag{5}$$