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Heat transfer enhancement of a periodic array of isothermal pipes

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ABSTRACT

We address the problem of two-dimensional heat conduction in a solid slab whose upper and lower surfaces are subjected to uniform convection. In the midsection of the slab there is a periodic array of isothermal pipes of general cross section. The main objective of this work is to find the optimum shapes of the pipes that maximize the Shape Factor (heat transport rate). The Shape Factor is obtained by transforming the periodic array of pipes into a periodic array of strips, using the generalized Schwarz–Christoffel transformation, and applying the collocation boundary element method on the transformed domain. Subsequently we pose the inverse problem, i.e. finding the shape that maximizes the Shape factor given the perimeter of the pipes. For large Biot number the optimum shapes are in agreement with the isothermal case, i.e. circular for sufficiently small perimeters/heat transfer, and elongated towards the surfaces of the slab for larger perimeters/heat transfer. Furthermore, for the isothermal case, we were able to discover a new family of optimum shapes for large thickness of the slab and large perimeters, which do not have their maximum width on the horizontal axis of symmetry. For small Biot number the optimum pipes are flatter than the isothermal ones for a given perimeter. The flatness becomes more apparent for larger perimeters. Most important, for large perimeters there exists a critical thickness which is characterized by maximum heat transfer rate. This is further investigated using the finite element method to obtain the critical thickness of a slab and the critical depth of the periodic array of circular pipes.

1. Introduction

In this work we address the problem of heat conduction in a solid slab embedded with a periodic array of isothermal pipes; the surfaces of the slab are subjected to convection with a uniform/constant convection heat transfer coefficient [1]. The shape of the pipes is assumed unknown and the main objective of this work is to find the shape that maximizes heat transfer. The particular configuration is a classical heat conduction problem that arises in connection with heating tubes, oil lines, steam distribution lines, underground electrical power-line transmission, laser sintering processes, in certain types of compact heat exchangers and solar cells [2–9].

A similar problem has been addressed by Fyrillas [10] where, however, the surfaces of the slab were assumed to be isothermal. When the slab is subjected to uniform convection, Fyrillas & Stone [11] showed that there exists a critical insulation thickness associated with a slab embedded with a periodic array of isothermal strips. Similarly, Fyrillas & Leontiou [12,13] also showed that there is a critical thickness associated with a fin that is subjected to uniform convection.

Following the analysis in Refs. [10,14–18], the physical domain is transformed into a rectangular channel using the generalized Schwarz–Christoffel transformation [19–22]. The heat transfer problem in the transformed domain is addressed numerically using the "singular" boundary element method [23–29].

As mentioned earlier, the main objective of this work is to pose and solve a Shape Optimization problem, i.e. an inverse design problem, where the objective function is the Shape Factor [1,30], i.e. the total heat transfer rate, and the variable of the optimization is

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Nomenclature

Bi	Biot number $= L\,h/k$
G	Green's function
h	one half the thickness of the slab in the complex domain
h	convection heat transfer coefficient W/(m^2 K)
H	one half the thickness of the slab in the physical domain (dimensionless)
k	thermal conductivity W/(m K)
L	dimensional distance between two consecutive pipes (period, m). Length-scale used for non-dimensionalization
P	one half the perimeter of the pipe (dimensionless)
S	shape factor (dimensionless)
T	temperature, K
x,y	coordinates of the physical plane (dimensionless)
z	complex coordinate of the physical plane
z_i	vertices in the physical plane

Symbols

α_i	turning angles divided by π
ξ, η	coordinates of the transformed domain
$w = \xi + i\eta$	complex coordinate of the transformed domain
w_i	image of z_i vertices in the transformed domain

Subscripts

i	related to the i-th vertex

Diacritic

$\wedge$	the variable is normalized with w_{N-1}

the shape of the pipe, which is parameterized though the parameters of the generalized Schwarz–Christoffel transformation. Hence, using the parameters of the generalized Schwarz–Christoffel transformation, the Shape Optimization problem is posed as a nonlinear programming problem (constrained nonlinear optimization [31]), which is solved numerically [32] to find optimum shapes that maximize heat transfer. We should point out that the application of the generalized Schwarz–Christoffel transformation confines the Shape Optimization to simply-connected domains. For general domains, one needs to consider a conformal transformation for multiply-connected domains [22] that might not be available. In such domains, although the direct problem can be solved in the physical domain using the boundary element method [23] the inverse problem, i.e. the shape optimization problem, would be intractable. It can be addressed by considering a more specific geometric parametrization and use boundary element methods if the governing PDE is the Laplace equation [33–36], or finite element methods for more general problems [37–40]. Although it is tempting to infer that conformal mapping techniques provide a natural basis for Shape Optimization problems associated with the Laplace equation in simply-connected domains, for general cases one needs to consider more general formulations [41–43].

The case of a single pipe in an infinite domain was treated in Ref. [15] where it was shown that the circular shape is the optimum shape for both maximization and minimization problems. In addition it was shown that: (i) the heat transport rate maximization problem, for a given perimetric length, is equivalent (dual) to the perimeter minimization problem for a given transport rate; and that (ii) the heat transport rate minimization problem, for a given area of the cross section, is equivalent (dual) to the area maximization problem for a given transport rate.

The duality of the shape optimization problems was also shown to apply for the case of a single isothermal pipe embedded in a slab (bounded domain), where the upper and lower surfaces of the slab are maintained at a constant temperature [16], while the slab is infinite in the horizontal direction. A circular shape is the optimum shape in the limit of small transport rates, i.e. the thickness of the slab is large. For larger transport rates, the optimum shapes tend to elongate towards the surfaces of the slab for the Shape Factor maximization problem, while it is elongated in the horizontal direction for the Shape Factor minimization problem. It is interesting to note that the optimum shape of the pipe does not extend beyond the half thickness of the slab for the Shape Factor minimization problem [44–46].

The case of a periodic array of isothermal pipes was treated in Refs. [10] and [18]. In the former work, both surfaces of the slab were assumed isothermal while in the latter, the lower surface was assumed adiabatic. In general the results suggest that, for the Shape Factor maximization problem, the optimum shapes are elongated towards the isothermal surfaces of the slab because this leads to a large temperature gradient due to the proximity of the pipe to the isothermal surfaces of the slab, hence a high transport rate is achieved. As far as the duality between the shape optimization problems is concerned (described in the previous paragraphs), for a periodic domain there is no rigorous prove of its existence. However, it has been justified through numerical simulations.

The existence of a critical thickness associated with a slab subjected to convection and embedded with isothermal strips [11], establishes that there is a significant difference between an isothermal slab and a slab subjected to convection. Hence, in the current work we investigate the optimum shape of the pipes when the surfaces of the slab are subjected to convection. In the next Section (§2), we describe briefly the numerical solution of the problem, i.e. the conformal mapping technique and the boundary element method. In Section §3 we pose and solve numerically the Shape Optimization problem of finding the optimum shape such that the heat transfer rate is maximized. In Section §4, using finite element simulations, we verify the existence of a critical thickness associated with a slab embedded with a periodic array of circular pipes when the slab is subjected to convection. In addition it is revealed that there exists a critical depth associated with pipes embedded in an insulated slab. We summarize our findings in the last Section.

2. Shape factor of a periodic array of isothermal pipes

The analysis of this section closely follows the definitions and notation outlined in Ref. [10], where the surfaces of the slab were assumed to be isothermal. In this work we assume that the surfaces of the slab are subjected to uniform convection with a uniform heat transfer coefficient (h), hence the work considered in Ref. [10] is the asymptotic limit of the present analysis for h $\to \infty$ (large Biot number, $Bi \to \infty$), i.e. strong convection.

Consider heat conduction due to a periodic array of isothermal (T_1) symmetric pipes of general cross section, embedded at the center of a solid slab. The temperature field is governed by the Laplace equation (Fig. 1). The upper and lower surfaces of the slab are subjected to convection with a constant convection heat transfer coefficient (h) and a constant far-field temperature T_∞ [1]. We non-dimensionalize lengths with the distance between two consecutive pipes (L), i.e. the period, and the temperature by subtracting T_∞ and dividing by the temperature difference $T_1 - T_\infty$. The dimensional analysis leads to the following definition for the Biot number $Bi = L\,$h$/k$, where k is the thermal conductivity. In addition,