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# A thermal non-equilibrium perspective on instability mechanism of non-isothermal Poiseuille flow in a vertical porous-medium channel

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ABSTRACT

We examine the influence of local thermal non-equilibrium state (LTNE) on the instability mechanism of non-isothermal Poiseuille flow in a vertical channel filled with porous medium. The temperature field of the solid porous matrix and the saturating fluid are governed by separate, but coupled, energy equations supplemented with three dimensionless LTNE parameters: inter-phase heat transfer coefficient ($H$), porosity scaled conductivity ratio ($\gamma$) and diffusivity ratio ($\Gamma$). The linear stability results show that the critical Rayleigh number ($Ra_c$) increases (decreases) with $H$ ($\gamma$), which indicates that the effect of $H$ ($\gamma$) is to stabilize (destabilize) the flow. The stabilizing impact of $H$ for fluid with low Prandtl number ($Pr$) becomes high when kinetic energy due to non-isothermal effect is lost to the basic flow. The energy loss to the basic flow increases on increasing (decreasing) $H$ ($\gamma$). Depending on the magnitude of all the studied parameters, three types of instabilities, namely: shear, mixed and buoyant have been observed. For relatively low permeable medium buoyant instability becomes the most dominant instability in the range of Prandtl number considered in this study.



## 1. Introduction

The mechanisms of heat transfer and fluid flow in porous medium, when the solid and fluid phases that constitute medium are in local thermal non-equilibrium (LTNE) state are expected to be different from the same under local thermal equilibrium (LTE) state [1,2]. To investigate the flow dynamics as well as heat transfer mechanism in such situation, a model of two medium treatment in the single phase flow, formulated by Carbonell and Whitaker [3], is well accepted [1]. In this approach, it is assumed that each phase is continuous and represented with an appropriate effective thermal conductivity.

In the present paper, the impact of LTNE state on the stability of Poiseuille flow in non-isothermal wall bounded vertical channel (i.e. fully developed mixed convection) is investigated. Mixed convection flow in vertical geometry is a problem of current interest and frequently encountered in various applications such as: packed-bed reactors, catalytic and chemical particle beds, solid matrix heat exchangers, packed bed re-generators, fixed-bed nuclear propulsion systems, and bio-fuel industry etc. In many such applications, the solid and fluid phases that constitute the porous medium are in local thermal non-equilibrium (LTNE) state. There are a number of published studies that are related to the present investigation. However, all of the analysis performed to date are based on the assumption that the medium is under LTE state. To gain a better perspective of results to be presented, we summarize their primary conclusions.

Based on the above applications some attempts have already been made by Bera and Khalili [4–6], Kumar et al. [7] and Chen [8] to investigate the stability characteristic of the non-isothermal Poiseuille flow in vertical porous medium channel.

Bera and Khalili [4] have investigated the instability mechanism of buoyancy assisted (stably stratified) Poiseuille flow when porous medium is hydro-dynamically as well as thermally anisotropic in nature. They have found that the least stable mode is two-dimensional and high media permeability destabilizes the flow. The energy analysis at critical level predicts three different types of instabilities (shear, mixed and buoyant instability) of the basic flow. Later on similar findings are also reported by Chen [8] while studying the stability of buoyancy-assisted mixed convection in the same geometry filled with an isotropic porous medium.

The stability of the buoyancy opposed Poiseuille flow is investigated for relatively low permeable medium by Bera and Khalili [5,6]. In Ref. [5], the influence of Prandtl number has been studied

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by considering Darcy number ($Da$) of the order of $10^{-5}$ and $10^{-6}$, and porosity as 0.4. They found that in general increasing of Prandtl number destabilizes the flow. For fluid having Prandtl number $\geq$0.7, two different types of instabilities: Rayleigh-Taylor (R-T) and buoyant instability were observed by them. The R-T instability mode was observed for relatively small values of Reynolds number, and corresponding critical value of $Ra$ was found as $-2.47/Da$. In order to understand the impact of permeability as well as porosity of the medium, a step is taken in the Ref. [6]. In this work, they considered a wide range of media-permeability that includes permeability as small as that of beach sand. For very low permeable medium the mode of instability was R-T type.

Later on, Kumar et al. [7] have examined the effect of drag forces as well as inertia on the stability of stably stratified flow in the vertical channel. They made a comparative study and found that the combined effect of form drag and convective term on the stability of the flow is more intensive than their individual effects, provided, the medium is highly permeable.

Using Darcy model, Barletta [9] has studied the stability of the mixed convection in a vertical porous channel with uniform wall heat flux boundary condition, where the basic flow does not satisfy the no slip condition.

Literature review reveals that the model based on LTE assumption, for convection in porous media, always is not necessarily a good approximation. At sufficiently large Rayleigh number or rapid heat transfer for high speed flow, it can be expected that the hot fluid stream penetrates well into relatively cold porous structures and hence in a representative elementary volume its temperature becomes sufficiently higher than that of the adjacent solid phase. So LTE assumption depends on the parameters of the problem (Kaviany [1], Vafai and Sozen [10], Al-Sumaily et al. [11]). Al-Sumaily et al. [11] investigated the effect of particle diameter of a packed bed of spherical particles on forced convection about an embedded cylinder numerically. They found that the agreement between experimental results [12] and numerical results under LTNE model is much better than the same between experimental results and analytical results [13] under LTE model.

Based on important applications (e.g., in computer chips via use of porous metal foams [2,14], in drying/freezing of foods [15,16], in microwave heating [17] in fuel cells [18], in nuclear reactor maintenance [19], in heat exchangers [20,21], and references therein) convection in porous media under LTNE state becomes an area of intense research. Different important works which utilize the LTNE theory are well documented in the literature (e.g., [11,22–24]). However, most of the studies are based on the assumptions that the flow is smooth and stable. Although, a fairly good number of articles have focused on thermal non-equilibrium impact on flow instability, but the majority (e.g., [25–29]) of these are related to Rayleigh Bénard convection in horizontal porous layer only. Compared to the work in horizontal porous layer, influence of LTNE state on flow transition in vertical geometry is limited to work of Rees [30], and Scott and Straughan [31]. The linear stability of natural convection, due to heating and cooling of side walls of the vertical channel, was investigated by Rees [30]. It was found that basic state of natural convection is always stable to disturbances of all wavenumbers at all values of the Darcy-Rayleigh number. In the case of convection due to differentially heated channel, Scott and Straughan [31] derived a threshold Rayleigh number by nonlinear energy stability theory below which convection will not occur.

From the above literature review it is clear that the stability characteristic under LTNE state has not been extended to mixed convection in channel. This motivates us to study the stability characteristic of non-isothermal Poiseuille flow in a vertical porous medium channel under LTNE state.

Precisely, there are two objectives. The first objective is to understand the impact of LTNE state on the instability boundary of non-isothermal Poiseuille flow. The second objective is to understand the role played by LTNE state on the instability mechanism in terms of (i) energy growth rate at and around the instability boundary point (critical point), (ii) energy spectrum at critical point.

The paper is organized as follows. The governing equations and basic flow are described in Section 2. The linear stability problem is formulated in Section 3. The related results that includes dependence of instability mechanism on energy spectra are given in Section 3. Finally, a brief summary and conclusion is made in Section 4.

## 2. Equations of motion and basic flow

The problem considered in this paper is mixed convection, driven by an external pressure gradient and buoyancy force, in a vertical channel of width $2L$ filled with porous medium. The wall temperature ($T_w$) varies linearly with $x$ as $T_w = T_o + Cx$, where $C$ is a positive constant and $T_o$ is the upstream reference wall temperature. The gravitational force is aligned in the negative $x$-direction, as shown schematically in Fig. 1. The temperatures of fluid and solid phases are accounted separately by two heat transport equations [23,24]. The Boussinesq approximation is assumed to be valid.

Furthermore, the medium is assumed to be homogeneous and isotropic in permeability. The momentum equation is written under the assumption of Brinkman-Forchheimer extended Darcy model [32]. Before proceeding further, a note is made regarding consideration of different terms in momentum equation.

Due to lack of unifying theory in transport in porous media to understand the transport phenomena in porous media the different models have been emerged based on empirical results (i.e., experimental data) as well as different theoretical approaches (i.e.,

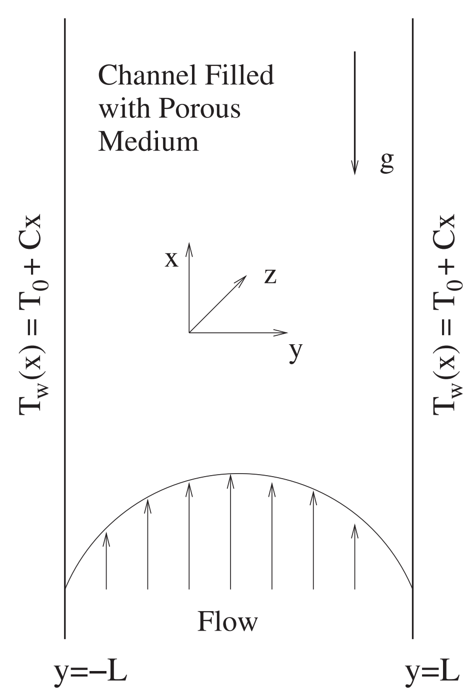


**Fig. 1.** Schematic of the physical problem.