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# Uncertainty in stretch extrapolation of laminar flame speed from expanding spherical flames

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**Abstract**

The present work investigated the uncertainties associated with the extrapolation of stretched flames to zero stretch in flame speed measurements using expanding spherical flames. Direct numerical simulations of time evolution of expanding spherical flames from a small ignition kernel to a propagating front with sufficiently large radius provide the relations between stretched flame speed and stretch rate that can be used to assess the uncertainty of extrapolation models. It is found that the uncertainties of flame extrapolation largely depend on the mixture Lewis numbers. While the uncertainty is minimized for stoichiometric $H_2$/air and *n*-heptane/air flames, the uncertainty can be as high as 60% for lean $H_2$/air mixtures, and 10% for lean and rich *n*-heptane/air mixtures. The present findings show that the weakly stretched flame assumption fails for lean hydrogen mixtures, and give a good explanation to the discrepancies between experiments and model predictions for $H_2$/air as well as the discrepancies between measurements of *n*-heptane/air using spherical and counterflow flames. A relation between extrapolation uncertainties and the product of Markstein number and Karlovitz number is provided, which can be useful for uncertainty quantification of future and existing measurements.


*Keywords:* Flame speed; Uncertainty; Extrapolation; Spherical flame; Stretch

## 1. Introduction

One of the most important global parameters of a combustion mixture is the propagation speed of the steady, one-dimensional, planar, adiabatic flame, namely the laminar flame speed, $S_u^0$. It is frequently used to validate combustion chemistry and advance our understanding of flame dynamics. Several flame configurations have been used to measure $S_u^0$, such as Bunsen flame, flat-burner flame, counterflow flame, and expanding spherical flame. Among these techniques, the expanding spherical flame is proven to be an effective method, especially at elevated pressures, that are typically not accessible using other configurations.

Despite of its simple geometry, expanding spherical flames are subjected to positive flame

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stretch, causing the flame speed to be modified by the nonequidiffusion of heat and mass characterized by non-unity values of the mixture Lewis number ($Le$) and preferential diffusion between fuel and oxidizer. Therefore, the stretch effect needs to be eliminated by extrapolating the measured flame speed to zero stretch, using relations between the local flame speed and stretch. Different stretch extrapolation relations and procedures have been used [1–4], and it is believed that the method for stretch extrapolation and the selection of experimental data for extrapolation is one of the major contributions of uncertainties. Recognizing that the extrapolation equations currently in use were all derived from asymptotic analysis based on various assumptions, such as one-step chemistry scheme, and weakly stretched flames, the validity and systematic uncertainties of these models under different conditions need to be adequately examined. Previous studies have compared the difference among the different models on experimental data [2,3,5]. However, little work is done on model validation against the "true" values.

It is also noted that the uncertainty in the stretch extrapolation of spherical expanding flames can be coupled with or magnified by the uncertainties caused by radiation [6–8], flow compression and confinement effects [4,6,9], especially at large flame radii. Unfortunately, due to the flame instability and limitation of imaging capability, few experiments data are available to understand the uncertainty in stretch extrapolation at large flame radius.

The present work aims to investigate computationally and experimentally the uncertainties of different extrapolation relations for flame speed measurement using the constant-pressure spherical flame method. The focus is on the extrapolation of stretched flame speed to zero stretch at large flame radius. In the present study, the uncertainty quantification is based on the computation results of both the 1-D planar flame and the expanding spherical flame using detailed chemistry without including radiation loss and absorption. The confinement effect in the computation is eliminated by using a large computation domain. New experiments were also conducted in order to compare with the computation results, and results with low extrapolation uncertainties are reported. Moreover, a criterion in terms of the product of Markstein number and Karlovitz number is provided to reduce the extrapolation uncertainty for practical spherical flame experiments.

## 2. Extrapolation equations

Table 1 summarizes all the five extrapolation equations evaluated in this study. The first one is a linear model based on stretch, denoted as LS. It was proposed by Wu and Law [1], and since then the large scatter in laminar flame speed measurements has been significantly reduced. Such a relation has been commonly used for flame speed measurement using expanding spherical flames. It is a first-order correction of the stretch effect, based on the assumption that $Le$ is near unity and the flame is weakly stretched. Therefore, some degree of uncertainty using this model in flame speed extrapolation is expected.

The second model is a nonlinear model suggested by Kelley and Law [2]. This relation was also based on weakly flame stretch and derived by Ronney and Sivashinsky [10]. It allows arbitrary $Le$, and hence is expected to be more general than the linear model and can be applied for arbitrary mixtures. For the past few years, this model has been used extensively for extrapolating laminar flame speeds from expanding spherical flames. Improved accuracy and performance have been demonstrated in [2,3,5]. Since the nonlinear model is based on quasi-steady flame propagation, we denote it as NQ in this study.

In addition to the NQ model, Chen and Ju derived a nonlinear flame speed and stretch relation of expanding spherical flames by including the effect of both strong stretch and general Lewis number [11] and suggested the third model, another nonlinear extrapolation equation in the limit of large flame radius [3]. Since the suggested model is also a linear relation between flame speed

Table 1
Extrapolation equations considered in this study. $S_b^0$ and $L_b$ are the unstretched laminar flame speed and the Markstein length with respect to the burned mixture, $S_b = dR_f/dt$, $K = (2/R_f)dR_f/dt$ and $\kappa = 2/R_f$ are the stretched flame speed with respect to the burned gas, the stretch rate and curvature of an expanding spherical flame, and $R_f$ is the flame radius.

| Model | Refs. | Notes | |
|---|---|---|---|
| LS | [1] | Linear model based on stretch | $S_b = S_b^0 - L_b K$ |
| NQ | [2] | Quasi-steady nonlinear model | $\left(\frac{S_b}{S_b^0}\right)^2 \ln\left(\frac{S_b}{S_b^0}\right)^2 = -\frac{2L_b K}{S_b^0}$ |
| LC | [3] | Linear model based on curvature | $S_b = S_b^0 - L_b\kappa = S_b^0 - \frac{2L_b}{R_f}$ |
| NE | [4] | Nonlinear model in expansion form | $\frac{S_b}{S_b^0}\left[1 + \frac{2L_b}{R_f} + \frac{4L_b^2}{R_f^2} + \frac{16L_b^3}{3R_f^3} + o^4\left(\frac{L_b}{R_f}\right)\right] = 1$ |
| N3P | – | Nonlinear model with 3 fitting parameters | $\frac{S_b}{S_b^0} = 1 - \frac{2L_b}{R_f} + \frac{C}{R_f^2}$ |

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