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An alternative to heat transfer coefficient: A relevant model of heat transfer between a developed fluid flow and a non-isothermal wall in the transient regime

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ABSTRACT

This paper deals with the relevant model that can be proposed for modelling interface heat transfer between a fluid and a wall for thermal boundary conditions varying in space and time. Usually, for a constant and uniform heat transfer (unidirectional steady-state regime), the problem can be solved through the introduction of the notion of a h heat transfer coefficient. This quantity, which is uniform in space and constant in time, links heat flux to a temperature difference (between the wall temperature T_w and an equivalent fluid temperature T_f, where h and T_f both depend on the system geometry) in a linear way.

The problem we consider in this work concerns the heat transfer between a dynamically developed steady-state fluid flow and a wall submitted to transient and non-uniform thermal excitations, for instance a steady-state flow over a flat plate submitted to a pulsed and space-reduced heat flux, or a steady-state flow in a duct stimulated by a periodic flux on its outer surface. More generally, we assume that this kind of thermal problem can be described by:

- one or several linear partial differential equations with their associated linear boundary and interface conditions;
- the coefficients of the homogeneous part of these equations do not depend neither on time nor on the coordinates in the direction parallel to the fluid/solid interface (they may depend on the coordinate in the normal direction);
- volume and surface sources (non-homogeneous part of the previous equations) that can depend on space and/or time.

We will show that the relevant representation for describing the interfacial heat transfer does not consist in defining a non-uniform and variable heat transfer coefficient $h(x,t)$, as done usually: the corresponding relationship is not really intrinsic because it depends on the thermal boundary conditions. An alternative approach is proposed here. It relies on the introduction of a generalized impedance $Z(\omega,p)$, which is a double integral transform of a transfer function $z(x,t)$ in the original space (x)/time (t) domain. This impedance function links heat flux and temperature difference through a convolution product (noted "⊗" here) rather than through a scalar product:

$$T_w(x,t) - T_f(x,t) = z(x,t) \otimes \varphi(x,t)$$

After a presentation of the generic problem, simple cases, with analytical solutions, will be presented for validation, such as a plug flow, in steady-state and transient regimes.

To conclude and show the interest of our approach, a comparison between a global approach and a numerical simulation in a more complex and less academic case will be presented.

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Nomenclature

a	thermal diffusivity, m^2 s^{-1}
e	thickness, m
f	function
F	Fourier transform
h	heat transfer coefficient, W m^{-2} K^{-1}
H	admittance in transformed domain ($H = 1/Z$)
i	imaginary unit
I	Bessel function
L	Laplace transform
Nu	Nusselt number
p	time Laplace variable, s^{-1}
Pr	Prandtl number
r	thermal resistance, m^2 K W^{-1} or space variable, m
Re	Reynolds number
s	space Laplace variable, m^{-1}
t	time variable, s
T	temperature, K
x, y	spatial variable, m
u, U	fluid velocity (x component), m s^{-1}
W	Whittaker function
WW	Whittaker function W
z	transfer function
Z	generalized impedance in transformed domain

Subscripts and superscripts

ˆ	quantities in space and time domain
0	sources of excitation
e	environment temperature
f	relative to the fluid
ϕ	uniform imposed heat flux
m	average temperature
ref	reference temperature
T	uniform imposed temperature
w	relative to the wall
∞	semi-infinite medium
*	reduced quantities

Greek symbols

δ	Dirac distribution
ϕ	heat flux density Laplace transformed
φ	heat flux density, W/m^2
Φ	transformed heat flux density
γ	heaviside distribution
λ	thermal conductivity, W m^{-1} K^{-1}
ν	cinematic viscosity, m^2 s^{-1}
θ	transformed temperature
ω	spatial frequency, m^{-1}

Symbols

$\otimes$	convolution product
•	dot product

1. Introduction

This paper deals with the relevant model that can be proposed for modelling interface heat transfer between a fluid and a wall for thermal boundary conditions varying in space and time. Usually for constant and uniform heat transfer (unidirectional steady-state regime), a heat transfer coefficient can be introduced:

$$\varphi = h\left(T - T_{\text{ref}}\right) \tag{1}$$

h being a constant in space and in time, T_{ref} represents an equivalent temperature, h and T_{ref} are functions of the system geometry.

The problem concerns the heat transfers between a steady-state fluid flow and a wall submitted to thermal solicitations in space and in time, for instance a steady-state flow over a flat plate submitted to localized heat flux stimulation, a steady-state flow in a duct stimulated by a periodic heat flux on its outer surface.

Extending the notion of heat transfer coefficient is a very old problem. As it is always possible to define a space and time varying heat transfer coefficient $h(x,t)$ so that $\varphi(x,t) = h(x,t)(T(x,t) - T_{\text{ref}})$, the problem is that this function is not very interesting because it depends not only on the fluid properties but also on thermal boundary conditions and solid properties (see for instance Refs. [1,2]). The idea of offering an "alternative to h" is not new and has been supported by Adiutori in 1974 [3]. This problem has been also widely discussed by different authors [4,5] but no real solution and interesting model have been proposed. More detailed references can be found in Ref. [6]. This problem is still of interest and even if some improvements have been made experimentally for evaluating thermal coupling between fluid and solid with the help of infrared thermography (see Refs. [7,8]) or by modelling (see Ref. [9]), few works have been carried out to propose a more relevant representation of the heat exchange between fluid flow and wall.

More generally, we assume that this kind of thermal problem can be represented by:

- one or several linear partial differential equations with their associated linear boundary and interface conditions;
- the coefficients of the homogeneous part of these equations do not depend neither on time nor on the coordinates in the direction parallel to the fluid/solid interface (they may depend on the coordinate in the normal direction);
- volume and surface sources (non-homogeneous part of the previous Equations) that can depend on space and/or time

The question is: what is the most adapted representation for modelling the heat transfer between a fluid and a non-isothermal wall in transient states?

An in-space and in-time varying heat transfer coefficient [$h(x,t)$] or generalized impedance [$Z(\omega,p)$]?

2. Generic problem

2.1. Problem

Let consider the following problem: a flow $u(y)$ between two walls in $y = 0$ and $y = e$ (see Fig. 1) submitted to a varying surface heat flux in x and in t (in this case walls are assumed as being neglected).

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{u(y)}{a}\frac{\partial T}{\partial x} + \frac{1}{a}\frac{\partial T}{\partial t} \tag{2}$$

$$\text{in } x = \pm\infty, \quad T = T_e \text{ and } \frac{\partial T}{\partial x} = 0 \tag{3}$$