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A mathematical model of a solar air thermosyphon integrated with building envelope

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ABSTRACT

A mathematical model comprising of analytical and two dimensional network procedures is proposed for predicting steady heat flow and heat transport in a solar heated thermosyphon integrated with building envelope. The detail mathematical solution method is presented for solving partial differential heat flow and transport equations by performing two dimensional energy balances at surface and air nodes through conjugate heat exchange and heat transport analysis of a solar thermosyphon. The mathematical solution procedure is devised for conduction and radiation heat exchange between surface nodes to improve the accuracy of traditional analytical solution for predicting buoyancy-induced mass flow rate of air flowing through a solar thermosyphon. The matrix inversion solution of mathematical model is unconditionally stable, with accuracy dependent on magnitude of conductance terms and number of nodes in the grid. Only Δy is chosen as aspect ratios ($\Delta x = L$, L/H and W/H) are defined by the geometry of the thermosyphon. The conduction and convection conductance terms are based on discretisation height Δy, thermal capacity conductance (mc_p) is based on air-gap length Δx, whilst integrated radiation conductance terms are based on both height Δy and width Δx of the grid. The proposed model has compared the results obtained from analytical method, two dimensional network method for a single set of environmental condition with given geometry of thermosyphon. The proposed model is validated with experimental results obtained from outdoor experimental setup comprising of thermosyphon based photovoltaic solar wall system.

1. Introduction

The present state of work for developing model for a thermosyphon is available for design of industrial thermosyphon with no exposure to solar heat flux. Most of the work in the literature is restricted to geometries based on experimental work of laboratories. The motivation of the paper is to present a detailed mathematical model for a steady state analysis of rectangular solar heated thermosyphon connected to a building. The mathematical model is interpreted from the view of physical thermal and fluid flow phenomenon occurring in the thermosyphon. In literature most of work is done on Trombe wall [1]. In the work on Trombe wall, the theoretical calculations for the temperature distribution of a Trombe wall are obtained by using a thermal network and compared the results with the experimental data [1]. Work is also performed on system modelling and obtaining operation characteristics [2]. Trombe wall with PV cells is also modelled [3].

The mathematical models in various scientific applications are solved by simultaneous partial differential equations. The numerical solutions are based on solving techniques such as Gauss–Jordan elimination in which large system of equations are reduced to obtain the simplified solution [5]. The solution methods for solving system of algebraic nodal equations are simplified with matrix computations. The unknown temperatures at nodes on discretised surfaces for a given system are determined with direct matrix solution methods. Despite availability of various numerical methods for solution of linear algebraic equations, there are many factors involved for pursuing the search for the analytical solutions. The analytical solutions on the other hand are useful in validation of numerical analysis because many parameters are kept constant in making assumptions during the development of a numerical method. Apart from this factor, analytical solutions provide sensitivity analysis so as to take into account the effect of parameters not taken into consideration during numerical analysis. However the non-availability of the closed-form analytical solutions for the partial differential equations has developed the need of numerical methods based on the discretisation principle [6].

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2. Operation of a solar air thermosyphon integrated with building envelope

The rectangular solar heated thermosyphon is integrated with the building through its well-insulated inner wall. The outer wall of a solar thermosyphon is exposed to uniform quasi steady-state solar heat flux. The cold fluid source in a solar thermosyphon is ambient air moving into the building hot-space sink through a solar thermosyphon. The schematic of a solar heated thermosyphon is illustrated in Fig. 1. The three different interacting temperature variables with only two space coordinates are used in heat flow analysis of a solar thermosyphon. The heat flow is a vector quantity associated with kinetic energy $\{m_k(v_k)^2/2\}$ of molecules in solid or fluid with motion from higher temperature region to lower temperature region. The origin, direction and magnitude of heat flow vectors are obtained by solution of simultaneous heat equations of thermal network for a given system. The Fourier's conduction law gives negligible heat flow value towards negative y-direction for air flowing through a solar thermosyphon because of its very low thermal conductivity. The procedure is useful to eliminate one temperature dependent conduction term in performing energy balance on air nodes.

The partial differential heat equation may contain up to four independent variables-three space coordinates and time. It is a linear partial differential equation in temperature variable for the case of constant thermo-physical properties. The boundary condition of radiation heat exchange at the enclosure surface of the walls is associated with long range distance phenomenon and conservation of energy is not applicable with in the control volume, but is applied in between all the composite surfaces observing net radiation exchange. The unique characteristics of the improved numerical solution method are: (i) inclusion of radiation exchange calculations using radiosity-irradiation method by assuming enclosure between outer and inner walls of a solar thermosyphon; and (ii) inclusion of conduction heat flow along y-direction for outer and inner walls of a solar thermosyphon.

The thermal network for a solar heated thermosyphon in conjunction with the building thermal system is associated with-(i) establishing heat travel and exchange paths; (ii) thermal design data-composition of thermal air and equipment/building construction materials-their equivalent heat capacities and conductances; (iii) environmental and climatic variables-prevailing wind direction and speed, solar irradiation, humidity, pressure and dry bulb temperature; (iv) feedback thermal set point control; and (v) load variation with periodicity and thermal storage. Simultaneous effect of above variables on the model is a complex phenomenon. Thermal network with large number of inter-dependent variables leads to arbitrary conditions solvable through stochastic models.

The affect of conjugate heat exchange and heat transport on temperature distribution in a solar thermosyphon is considered so as to improve the accuracy of numerical solution. The climatic and thermal design data is kept constant in steady heat flow analysis of a solar thermosyphon. The single climatic variable of ambient air temperature, solar irradiation and building zone air temperatures are known constants in the analysis.

3. Mathematical physics, analysis and geometry

The paper has presented the detailed mathematical physics, analysis and geometry for a solar heated thermosyphon connected to a building as illustrated in Fig. 2. The heat exchange analysis is carried out on the geometry of a solar thermosyphon with discretisation of its total covered volume into surface and air nodes located by formulation of control volumes. A geometrical method is illustrated for conduction and radiation heat exchange between surface nodes to improve the accuracy of traditional analytical solution for predicting buoyancy-induced mass flow rate of air flowing through a thermosyphon. As is illustrated in Fig. 2, a solar thermosyphon is placed along the y-axis with y = 0 near the bottom end of the system boundary and y = H near the top end of the system boundary. The solar thermosyphon is rectangular in cross-section with width W in z-direction and air-gap length, L in x-direction. The thermal conductivities of outer wall and inner wall are assumed to be constant along with their dimensions-L, W and H. The inner wall is well-insulated with thermal conductance u_i. The outer wall is of good thermal conductance (u_o) for conducting heat flux of solar irradiation. The heat transfer between building space and well-insulated inner wall is assumed to be nil. The heat transfer between side walls of length L, and height H and surrounding zone is also assumed to be nil. The air passage for a solar thermosyphon is connected with the building space through a damper operating system. The physical domain of a solar thermosyphon is analysed as a parallel-plate channel.

The mathematical analysis for the heat flow consists of discretisation of volume of thermosyphon into the small volume element **V** between four cross-sectional planes P_{1x}, P_{2x}, P_{1y} and P_{2y}. The planes P_{1x}, P_{2x} are perpendicular to the y-axis with plane P_{1x} located at y and plane P_{2x} located at $y + \delta y$. The planes P_{1y}, P_{2y} are perpendicular to the x-axis with plane P_{1y} located at $x = t_o$ and P_{2y} is located at $x = t_o + L$. Further the planes P_{1x} and P_{2x} are having

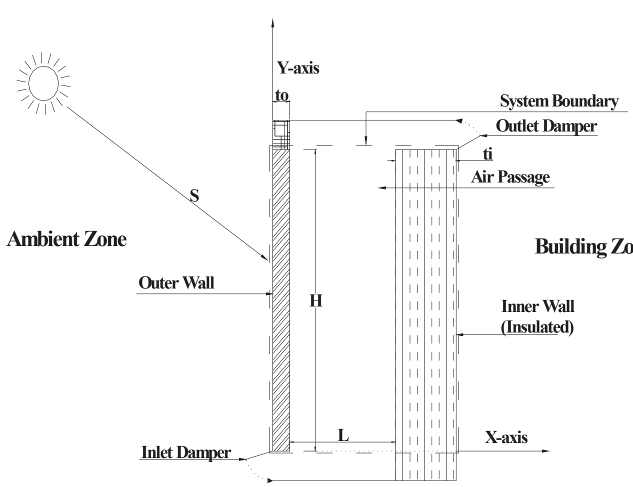

Fig. 1. Schematic of a solar heated thermosyphon-open rectangular type.

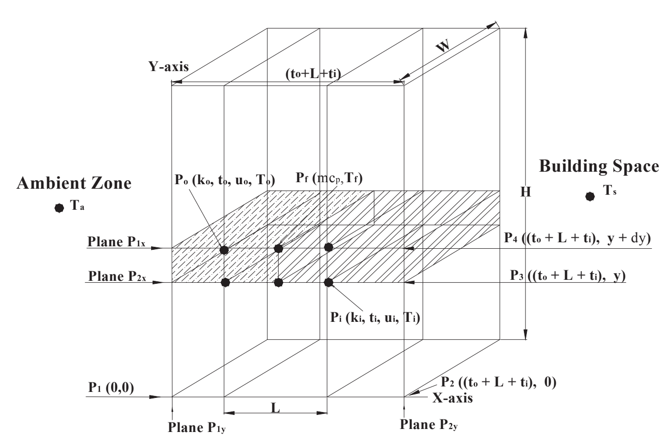

Fig. 2. The system coordinates: Nodes and cell faces in a control volume.