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Sine-square embedded fuzzy sets versus type-2 fuzzy sets

Mehmet Karakose*, Hasan Yetis, Erhan Akin

Firat University, Computer Engineering Department, 23119 Elazig, Turkey

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ABSTRACT

The uncertainty is an inherent part of real-world applications. Type-2 fuzzy sets minimize the effects of uncertainties that cannot be modeled using type-1 fuzzy sets. However, the computational complexity of the type-2 fuzzy sets is very high and it is more difficult than type-1 fuzzy sets to use and understand. This paper proposes sine-square embedded fuzzy sets and gives a comparison with type-2 and nonstationary fuzzy sets. The sine-square embedded fuzzy sets consist of type-1 fuzzy sets and the sine function. The footprint of uncertainty in the type-2 fuzzy sets is provided with amplitude and frequency of sine-square function in the proposed algorithm. The proposed sine-square embedded fuzzy sets are much simpler than the type-2 fuzzy sets and the nonstationary fuzzy sets. Two control applications that are chosen as position control of a dc motor and simulation of human lifting motion using five-segment human model are carried out to demonstrate the effectiveness of the proposed approach.

1. Introduction

The concept of uncertainty that is fully connected with the concept of information appears in many areas such as reasoning, decision making, and control systems. The uncertainty is a result of some information deficiency that may be imprecise, incomplete, not fully reliable, or vague. The fuzzy reasoning that uses to represent uncertainty by numbers in the range [0, 1] allows handling much of this uncertainty. It is known that much of the reasoning performed in practical problems involves linguistic information. Therefore, fuzzy reasoning mechanisms based on type-1 fuzzy sets have been proposed by Zadeh in 1965 [1,2]. The concept of the fuzzy set theory was introduced as an extension of the classical notion of set permitted the gradual assessment of the membership of element in a set. The fuzzy set theory has been applied in a wide variety of areas such as artificial intelligence, control systems, modeling, signal processing and expert systems [2–4]. The fuzzy set theory has been investigated in order to represent uncertainty in measurements. A classical set and a membership function for this set can be given as shown in Fig. 1.

A type-1 fuzzy set can be represented as a set of pairs of x and its degree of membership function $\mu_F(x)$.

$$F = \{(x, \mu_F(x)) \mid x \in X\} \tag{1}$$

The degree of membership to a type-1 fuzzy set indicates the uncertainty. $\mu_F(x)$ for classical two-valued sets is either 0 or 1, but $\mu_F(x)$ for type-1 fuzzy sets takes on values in the range [0, 1] as shown in Fig. 2(a) and (b). The membership functions for type-1 fuzzy sets can be of any type or shape as determined by experts such as triangular, trapezoidal, Gaussian, exponential [2]. The type-1 fuzzy systems as a formal logic system are very useful in representing human knowledge in a domain of application and in reasoning. A type-1 fuzzy system based on type-1 fuzzy sets consists of four main components as shown in Fig. 2(c).

A correct representation of uncertainties in measurements resulted from systematic errors, random effects, and natural dispersion is crucial in many applications. When a measurement or information is uncertain, it is difficult to determine its exact value and type-1 fuzzy sets make more sense than classical sets. However, the type-1 fuzzy sets cannot directly model such uncertainties, because they are characterized by crisp membership functions [5]. The type-2 fuzzy sets are very useful for modeling and minimize the effect of uncertainties that can occur because of data, measurements, rule consequents and meanings of the words in the rules [5,6]. Type-2 fuzzy sets provide second order uncertainties for real-world problems while type-1 fuzzy sets provide a formal mathematical method for dealing with vague or imprecise [7–9]. A type-2 fuzzy set can be characterized by a type-2 membership function $\mu_A(x,u)$ [6].

$$A = \{((x,u), \ \mu_A(x,u)) \mid \ \forall x \in X, \ \ \forall u \in J_x \subseteq [0,1]\} \tag{2}$$

The uncertainties can be modeled by a blur membership function called as the footprint of uncertainty (FOU) that is bounded by an upper membership function and a lower membership function as shown in Fig. 3(a). Type-2 fuzzy logic appears as interval and generalized type-2 fuzzy logic. The interval type-2 fuzzy logic is simpler than other as

* Corresponding author.
E-mail addresses: mkarakose@firat.edu.tr (M. Karakose), h.yetis@firat.edu.tr (H. Yetis), eakin@firat.edu.tr (E. Akin).

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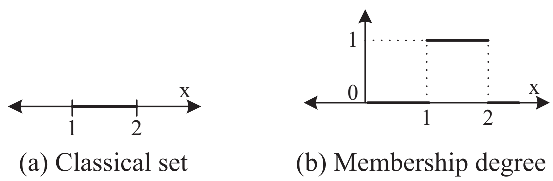

Fig. 1. The membership degree for classical sets.

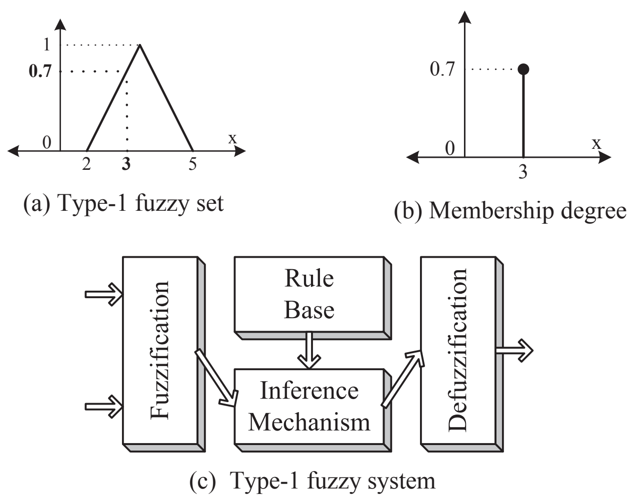

Fig. 2. The membership degree for type-1 fuzzy sets and structure of a type-1 fuzzy system.

shown in Fig. 3(b)–(d) shows type-2 fuzzy sets. Type-2 fuzzy sets have three-dimensional membership functions. Therefore, type-2 fuzzy systems that use type-2 fuzzy sets are more complex than type-1 fuzzy systems. The schematic diagram of a type-2 fuzzy system based on type-2 fuzzy sets is shown in Fig. 3(e). The type-2 fuzzy system has a type-reduction stage according to type-1 fuzzy systems. A type reducer performs a centroid calculation on type-2 fuzzy sets, which leads to a type-1 fuzzy set. Many researchers in the past decades have been investigated type-2 fuzzy sets and their properties [10–13].

In the recent literature, many studies have been presented to overcome the problem of computational complexity of type-2 fuzzy systems [14–22]. The main focus of many of these studies is reducing the computational complexity of type reduction step which is the main reason for high computational burden of type-2 fuzzy systems. So, to enhance the performance of Karnik-Mendel, which is a known type reduction algorithm, new variants of this algorithm are used [14–17]. Also, new methods were investigated to actualize the type reduction process with less computational efforts. One of these studies done by Li [18]. Li proposed an efficient type reduction method based on the interval analysis. He compared his method with COS type-reduction method, and he reported that their method is much more efficient than COS. Coupland [19,20] proposed a geometric approach to fuzzy systems. This approach represents a type-1 fuzzy set as a two-dimensional geometric object and the type-2 interval fuzzy sets are modeled as two type-1 fuzzy sets with geometric representation. Wu has compared the type reduction and defuzzification methods in the literature for type-2 fuzzy systems in his work [21,22]. In his study, the type reduction methods have been studied in three different categories in terms of the computational cost of the fuzzy systems and detailed comparison results are given. Li proposed a closed-form defuzzification method based on Nie-Tan operator for interval type-2 fuzzy systems [23]. By using a closed-form solution, the computational complexity of type reduction step is reduced. Hamrawi defined the alpha-cut decomposition theorem for type-2 fuzzy systems [24]. He proposed a new representation similar to the alpha-cut representation which is in type-1 fuzzy sets. In another study, Wu proposed a simplified type-2 fuzzy logic controller for a coupled-tank liquid level control process that provides computational savings in real time [25]. The main idea in this study to replace some critical type-1 fuzzy sets by type-2 fuzzy sets in a fuzzy system. In studies of Mendel [26,27], computation of the centroid for a type-2 fuzzy set that is symmetrical is reduced by%50. It proposes an approach establishing a small set of terms that let us easily communicate about type-2 fuzzy sets and a representation to derive formulas for union, intersection, and complement of type-2 fuzzy sets. Liu [28] has proposed a method to decrease the computational complexity from exponential into linear for the type-2 fuzzy systems. In the study of Dalalah [29], a piecewise parametric polynomial approach is proposed to model fuzzy sets using a recursive formulation that the mathematics generated by the approach is consistent and can be simply generalized to standard applications.

Garibaldi [30] has proposed nonstationary fuzzy sets, which uses type-1 fuzzy sets to model uncertainties similarly to the proposed method. A nonstationary fuzzy set is a collection set of type-1 fuzzy sets and was obtained by repeatedly generating a membership function. The nonstationary fuzzy set can be represented as follows [30].

$$
\begin{aligned}
&F = \int_{t\in T}\int_{x\in X} \mu_A(t,x)/x/t \\
&\mu_A(t,x) = \mu_A(x,p_1(t),\dots,p_m(t)) \\
&p_i(t) = p_i + k_i\, f_i(t) \quad i = 1,2,\dots,m
\end{aligned} \tag{3}
$$

where μ_A:T × X → [0,1] is a nonstationary membership function, X is s universe of discourse, T is set of time points t_i and f:T → $\Re$ denote a perturbation function that is varied in time to each parameter multiplied by a constant. The perturbation function can be periodic, pseudorandom and differential time-series functions [30]. A triangle nonstationary fuzzy set and a nonstationary fuzzy system are shown in Fig. 4(a)–(c). Only five membership functions for a nonstationary fuzzy set have been shown in Fig. 4(a). More membership functions such as 50, 100, or 1000 should use for desired response. The nonstationary fuzzy inference systems do not have some difficulties of type-2 fuzzy systems, but a nonstationary fuzzy system has still computational cost resulted from the repetition of type-1 fuzzy inference [31].

In most of the studies in the literature either the type-2 fuzzy systems have been tried to be simplified or various mathematical representations have been given for fuzzy sets. The most of the studies focused on reducing the computational cost of type reduction step in type-2 fuzzy systems. So, there are still drawbacks such as computational complexity arises from type reduction process. Unlike the studies in the literature, the paper is focused to represent the FOU by type-1 fuzzy system instead of type-2 fuzzy system. The aim of this paper is to develop an efficient fuzzy set for the fuzzy inference systems based on sine-square function. The main idea of proposed sine-square embedded fuzzy sets is derived from a bounded region termed the FOU of type-2 fuzzy sets. With the help of the proposed sine-square embedded fuzzy sets, the FOU can be modeled by type-1 fuzzy systems simply. Examination of the effects of the proposed sine-square embedded fuzzy sets on fuzzy logic systems in two examples has shown that the proposed approach proves the accuracy and computational complexity.

The rest of the paper is organized as follows: Section 2 presents the proposed sine-square embedded fuzzy sets. Section 3 gives simulation results with two problems. Finally, conclusions are explained in Section 4.

2. Sine-square embedded fuzzy sets

The uncertainty can be defined as the possibility of error or knowledge incompleteness that exists as a result of inherent deficiencies and having less than total information about environments or systems. We cannot generally avoid uncertainties in real-world problems. The sources of the uncertainty can be given with system parameters, measured data, randomness related to natural system, possibility of having multiple outcomes for systems, or system modeling.