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Non-Fourier heat conduction in a sandwich panel with a cracked foam core

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ABSTRACT

Crack formation in cellular solids is often triggered by the presence of flaws caused by the manufacturing process that is used to build them. In thermal management applications, an imperfection embedded in the core of a sandwich panel might have a detrimental thermal impact that is often challenging to predict. This paper focuses on a theoretical study of non-Fourier heat conduction in a sandwich panel with a cracked foam core. With the aim of exploring the thermal response of a panel with porous core and skins made of a single material, we examine the role of crack position, relative density of the foam core, and other geometric parameters of the panel. Based on the assumption that the crack in the core is thermally insulated, i.e. no heat flux can pass through, we obtain the temperature distribution and heat flux intensity factor in the time domain via Fourier and Laplace transforms. The results are visualized in maps that show the influence of the size of the skin relative to that of the foam core and the crack location as well as the antagonist impact that the relative density of the foam core has on the maximum temperature and heat flux. The method presented here can be used to tailor the thermal response of sandwich panels.

1. Introduction

Sandwich panels consist of two stiff and strong skins, or faces, bonded to a core [1]. While the former are typically metal or polymer/fiber-reinforce composites, the core has generally a low-density cellular architecture, such as that in metallic honeycombs, polyurethane foams, and other porous materials. The function of the core is to not only increase the bending stiffness of the panel and its resistance to buckling loads [1,2], but also satisfy multi-functional requirements, such as fluid permeability, energy absorption, and acoustic damping among others [3,4]. Low weight, structural efficiency, and multifunctionality, are desirable characteristics often sought after in aircrafts, automobiles, ships, and sport equipment. Usually, the cellular architecture of the core can be either random, such as in foams, or periodic, such as in corrugated (prismatic) metals and textile/truss cores [4], but it might be also a mix of them. In general, what controls the multifunctional response of a sandwich panel is the properties of the skin and the core. The thickness of the former, besides its material attributes, is a critical design parameter, whereas the relative density of the latter, i.e. the core, together with cell topology and nodal connectivity, governs the thermal, acoustic, and other responses of the sandwich panel.

Due to the increasing use of cellular materials in multifunctional applications, the study of their multiphysics has been the object of intense research. A wealth of studies is available in the literature, each dealing with either the theoretical and/or experimental aspects. On the prediction side, for instance, several homogenization schemes applied to cellular materials have been used to predict heat conduction and mass transport [5–8], electromagnetic permeability [9], as well as their mechanical responses [10–15]. On the testing side, the mechanical [16,17], thermal [18,19], and thermomechanical [20–22] responses of a sandwich panel with either a lattice or foam architecture have been measured for the use in load-bearing, biomedical, and thermal management applications.

In structural applications, the presence of a crack, either within the core or at the interface with the skins, can utterly drop the

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strength of the panel [23]. The magnitude of this reduction, as well as its associated evolution of fracture, have been studied by using standard concepts of continuum and fracture mechanics [24,25]. Whereas some studies have mainly examined the nucleation and propagation of a crack in a cellular material [9,26], others have focused on the cellular core in a sandwich panel, in particular its failure modes, such as the formation of an inter-layer crack and skin delamination [17].

In thermal management applications, a crack that forms in a sandwich core causes overheating its vicinity. The physics of this phenomenon poses some challenges for both the theory and experiments that are required to understand it [27–29]. First investigations back to 1965 examined a solid material containing a crack, where the singularity of the heat flux was obtained in steady state for an infinite medium with collinear cracks [30]. To quantify thermal energy accumulated around a macro-crack tip, Tzou [31,32] first introduced the intensity factor of the temperature gradient and then analyzed the effect of the thermal properties on the singularity exponent of the temperature gradient. More recently, the influence of an arbitrarily oriented crack on the heat conduction of a functionally graded (FG) medium has been elucidated and the discontinuity of the heat flux has been obtained via the heat flux intensity factor (HFIF) [33]. Similar to the concept of the stress intensity factor measuring the stress concentration in the vicinity of a crack tip, HFIF represents the intensification scale of the heat flux at the crack tip [34]. A larger value of HFIF implies a higher heat flux and a severer thermal condition around the crack tip. While the aforementioned studies pertain to a crack in a solid material, transient heat conduction analysis of a cellular material containing a crack is at its infancy and limited to Fourier heat conduction [35,36].

Fourier heat conduction is the conventional approach that has been extensively used to study heat conduction in crack problems. Although well established, one limitation of this theory is its accuracy, which is deficient for very low temperature and short-pulse thermal heating in micro temporal/spatial scale [37,38]. As a result, non-Fourier heat conduction schemes have been proposed, among which the simplest is that of Cattaneo–Vernotte (C–V) [39,40]. Another refinement, introduced by Tzou [41] to better describe heat conduction in a transient case, resorts to a dual-phase-lag (DPL) that accounts for the microscale temporal and spatial effects of the heat transport. The DPL model has been used to investigate heat conduction in layered composites [42], interface bonding of dissimilar materials [43], biological tissues [44], and FG materials [45–48]. Recently, non-Fourier theories have been also applied to solid media containing a crack, and to study the transient heat conduction in a cracked half-space [49], a bilayered composite with a penny-shaped interfacial crack [50], as well as a cylinder with an embedded/edge circumferential crack [51]. It has been observed that the phase-lags of heat flux and temperature gradient have a significant impact on the intensity factors of a cracked medium. Furthermore, DPL theory applied to a sandwich panel subjected to a thermal shock has the potential to capture the thermal wave propagation and the overshooting of the transient temperature induced by the thermal impact. Another advantage of using the DPL theory is that microstructural interactions occurring during the heat transport process and fast transient phenomena of thermal waves can be studied via the phase-lag of the heat flux. Despite these pros, no work exists in the current literature that uses non-Fourier heat conduction to understand the transient thermal response of a sandwich panel with a cracked core.

This paper aims at studying the disturbed temperature field of a sandwich panel, containing an insulated crack in its core that is made of foam. Both skins and core are assumed to be made of a single material, and the overall panel assumed to be rigid with no deformation occurring under transient thermal loadings. We use a DPL heat conduction model (Section 2) with the effective thermal properties of foams (Section 3) for thermal analysis. In Section 4, the two-dimensional temperature field is obtained in Laplace domain by Fourier transform and by solving singular integral equations. We also introduce HFIF to recognize the singularity of the heat flux field at the crack tip. The role of phase-lags, relative density, skin thickness, and crack position is examined in Section 5 with respect to the transient temperature field and HFIF, and the results are illustrated in the form of design charts [52–54].

2. Problem definition

We examine a sandwich panel with a foam core containing a thermally insulated crack, assumed here as a gap of length $2c$ (Fig. 1) [55]. The sandwich consists of: (1) an upper skin with thickness $h_3 - h_1$, (2) foam core of thickness $h_1 + h_2$, and (3) lower skin of thickness $h_4 - h_2$. The material properties of both skin and core do not change. The sandwich panel is initially at temperature T_∞, before applying a sudden temperature rise at the top, T_a, and the bottom, T_b, of the skins. Temperature rises occur via a Heaviside or step function $H(t)$, which is zero for negative values of t and is unity for non-negative values of t. In this investigation, since the solid skins and foam core are made of one material, there is a unique coefficient of thermal expansion, which does not change for the relative density range of the foam core. The result is that no residual stresses can develop at the interface between the skins and foam core with a perfect bond.

The DPL model for heat conduction can be described as:

$$\mathbf{q}(\mathbf{r}, t + \tau_q) = -k\nabla T(\mathbf{r}, t + \tau_T) \tag{1}$$

where $\mathbf{q}$, $\mathbf{r}$, t, k, and T are the heat flux vector, position vector, time, thermal conductivity, and temperature, respectively; ∇ represents the gradient operator and τ_q and τ_T are phase-lags of heat flux and temperature gradient, respectively. We use the Taylor series expansion (Eq. (1)) up to the second-order for τ_q, and up to the first-order for τ_T, which leads to the wave-like DPL model [45]

$$\left(1 + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right)\mathbf{q} = -k\left(1 + \tau_T\frac{\partial}{\partial t}\right)\nabla T \tag{2}$$

The energy conservation equation in the absence of internal heat generation is written as

$$-\nabla\cdot\mathbf{q} = \rho c_V\frac{\partial T}{\partial t} \tag{3}$$

where ρ and c_V are, respectively, mass density and specific heat capacity. Eliminating $\mathbf{q}$ in Eqs. (2) and (3) leads to the following hyperbolic differential equation:

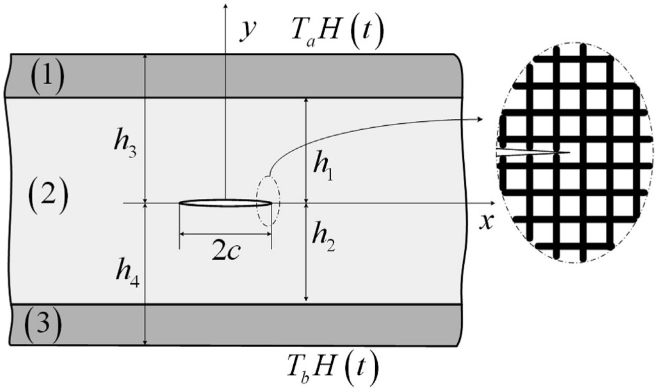

Fig. 1. A mono-dimensional sandwich panel with a cracked foam core.