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Double-diffusive Hadley–Prats flow in a porous medium subject to gravitational variation

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ABSTRACT

A model for double-diffusive porous Hadley–Prats flow under the effect of a variable gravity field is explored. Both linear instability and nonlinear stability analyses are performed to assess the suitability of linear theory to predict the destabilisation of the flow. The resultant eigenvalue problems for both theories are solved using shooting and fourth order Runga–Kutta methods for various flow governing parameters. The results indicate that both the linear and nonlinear results undergo quantitative changes subject to horizontal thermal and solutal gradients along with the presence and absence of gravity and mass flow variations. For certain parameter ranges there are also substantial regions of potential subcritical instability, such that instabilities may arise before the linear stability threshold is reached.

1. Introduction

Thermosolutal convection in a fluid-saturated porous media has received much attention during the last few decades, due to its wide ranging applications in multiple fields such as underground energy transport, cooling of nuclear reactors, food processing, underground storage of waste materials and oil recovery [1–4]. The effect of gravity in flows through porous medium has also attracted considerable interest [7–10], with a range of possible natural settings [5,6]. However, limited research has been conducted on natural convention in the presence of inclined gradients with a mass flow effect. The available studies have also not tended to include nonlinear theory [19]. Nield [11] explores mono-diffusive convection in a porous layer with inclined temperature gradients and a mass flow effect, with an extension into both temperature and concentration variations are studied by Nield et al. [12] and horizontal mass flow is analysed by Manole et al. [13]. A comprehensive background on heat convection in porous media may be found in Nield and Bejan [17].

Linear instability theory gives sufficient conditions for a perturbation of the basic state solution to be unstable. However, this approach provides limited information on the behaviour of the nonlinear system. The instability could then potentially occur prior to the thresholds predicted by the linear theory being reached. Establishing sufficient conditions for all perturbations to be asymptotically stable through the use of nonlinear stability theory [19] is therefore crucial in establishing the regions of potential subcritical instability. Examples of the utilisation of nonlinear theory can be found in Kaloni et al. [14,15], which specific application to the mass flow effect in Kaloni et al. [16].

In this article, we examine the effect of gravitational variation (cf. [10,16]) in the presence of mass flow by deriving both linear instability and nonlinear stability thresholds. The numerical solutions of the linear and nonlinear theories constitute generalized eigenvalue problems, which are evaluated using Shooting and Runga–Kutta methods for various modes of instability.

2. Governing equations

Let us consider a shallow horizontal fluid saturated porous layer confined between two fixed plates separated at a distance d, where the z^*-axis is vertically upwards and there is a net flow along the direction of x^*-axis with magnitude Q^* as shown in Fig. 1. The vertical concentration and temperature differences across the boundaries are ∇S and $\nabla\theta$, respectively. Furthermore, we define the horizontal components of solutal and thermal gradients to be

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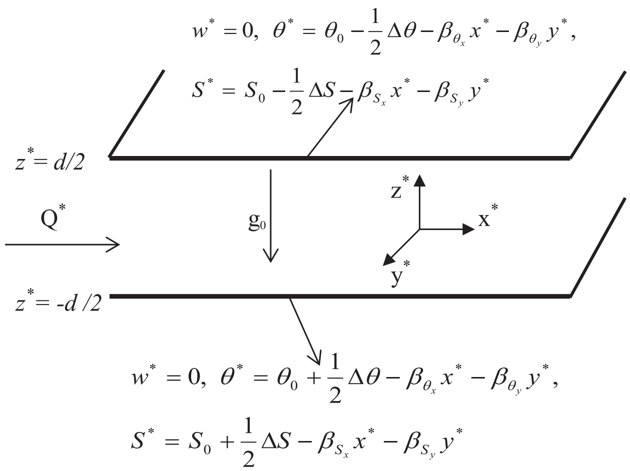

Fig. 1. Schematic diagram of the physical system.

$(\beta_{S_x},\ \beta_{S_y})$ and $(\beta_{\theta_x},\ \beta_{\theta_y})$, respectively. We assume that the Oberbeck-Boussinesq approximation is valid, hence, the flow in the porous medium is governed by the Darcy law (cf. Kaloni and Qiao [15]), and the conservation of energy and mass, the governing equations are given by

$$\nabla^* \cdot \mathbf{q}^* = 0, \tag{1}$$

$$\frac{\mu}{K}\mathbf{q}^* + \nabla^* P^* - \rho_f^* g = 0, \tag{2}$$

$$\phi\left(\frac{\partial S^*}{\partial t^*}\right) + \mathbf{q}^* \cdot \nabla^* S^* = D_m \nabla^{*^2} S^* \tag{3}$$

$$(\rho c)_m\left(\frac{\partial \theta^*}{\partial t^*}\right) + (\rho c_p)_f \mathbf{q}^* \cdot \nabla^* \theta^* = k_m \nabla^{*^2}\theta^*, \tag{4}$$

where (1) is the incompressibility condition. Here $\mathbf{q}^* = (u^*, v^*, w^*)$ is the velocity, P^* is the pressure, S^* is the concentration, θ^* is the temperature, μ is the viscosity, D_m is the solutal diffusivity and k_m is the thermal conductivity. K and ϕ are the permeability and porosity of the medium, respectively. The subscripts f and m refer to the fluid and porous medium, respectively. c_p and c are the specific heats of the fluid and solid components, with ρ_0 denoting the reference density. The density ρ_f^* of the fluid, and acceleration due to gravity g are given by

$$\rho_f^* = \rho_0[1 - \gamma_S(S^* - S_0) - \gamma_\theta(\theta^* - \theta_0)], \quad g = -g_0(1 + \gamma^* z^*)\mathbf{k}, \tag{5}$$

where $\mathbf{k} = (0,0,1)$, γ_S and γ_θ are the solutal and thermal expansion coefficients in the porous medium, and g_0, S_0 and θ_0 are the reference values of acceleration due to gravity, concentration and temperature, respectively. We denote γ^* to be the gravity parameter. The boundary conditions are given by

$$w^* = 0, \quad S^* = S_0 - \frac{1}{2}(\pm\Delta S) - \beta_{S_x} x^* - \beta_{S_y} y^*,$$
$$\theta^* = \theta_0 - \frac{1}{2}(\pm\Delta\theta) - \beta_{\theta_x} x^* - \beta_{\theta_y} y^* \quad \text{at } z^* = \pm\frac{1}{2}d. \tag{6}$$

which are discussed in Kaloni et al. [16]. Following the scalings of Weber [18], Nield [11] and Kaloni and Qiao [16] we define

$$x = \frac{x^*}{d},\ y = \frac{y^*}{d},\ z = \frac{z^*}{d},\ t = \frac{\alpha_m t^*}{ad^2},\ \mathbf{q} = \frac{d\mathbf{q}^*}{\alpha_m},\ P = \frac{K(P^* + \rho_0 g z^*)}{\mu\alpha_m},$$
$$\theta = \frac{R_z(\theta^* - \theta_0)}{\Delta\theta},\ S = \frac{S_z(S^* - S_0)}{\Delta S},\ \gamma = \frac{\alpha_m \gamma^*}{Ad^2},\ Q = \frac{dQ^*}{\alpha_m}, \tag{7}$$

where

$$\alpha_m = \frac{k_m}{(\rho c_p)_f},\quad a = \frac{(\rho c)_m}{(\rho c_p)_f},\quad S_z = \frac{\rho_0 g \gamma_S K d \Delta S}{\mu D_m},$$
$$R_z = \frac{\rho_0 g \gamma_\theta K d \Delta\theta}{\mu\alpha_m}. \tag{8}$$

here, S_z and R_z denote the vertical solutal and thermal Rayleigh numbers, respectively. The horizontal solutal and thermal Rayleigh numbers are defined as follows

$$S_x = \frac{\rho_0 g \gamma_S K d^2 \beta_{S_x}}{\mu D_m},\quad S_y = \frac{\rho_0 g \gamma_S K d^2 \beta_{S_y}}{\mu D_m},$$
$$R_x = \frac{\rho_0 g \gamma_\theta K d^2 \beta_{\theta_x}}{\mu\alpha_m},\quad R_y = \frac{\rho_0 g \gamma_\theta K d^2 \beta_{\theta_y}}{\mu\alpha_m}, \tag{9}$$

with $Le = \frac{\alpha_m}{D_m}$ being Lewis number. Under these dimensionless variables, the governing Eqs. (1)–(4) yield

$$\nabla \cdot \mathbf{q} = 0, \tag{10}$$

$$\mathbf{q} + \nabla P - \left(\frac{1}{Le}S + \theta\right)(1 + \gamma z)\mathbf{k} = 0, \tag{11}$$

$$\left(\frac{\phi}{a}\right)\frac{\partial S}{\partial t} + \mathbf{q} \cdot \nabla S = \frac{1}{Le}\nabla^2 S, \tag{12}$$

$$\frac{\partial\theta}{\partial t} + \mathbf{q} \cdot \nabla\theta = \nabla^2\theta, \tag{13}$$

with boundary conditions

$$w = 0, \quad S = -\frac{1}{2}(\pm S_z) - S_x x - S_y y,$$
$$\theta = -\frac{1}{2}(\pm R_z) - R_x x - R_y y \quad \text{at } z = \pm\frac{1}{2}. \tag{14}$$

From Eqs. 10–13 we observe that all variations of the solutal and thermal Rayleigh numbers appear in the boundary conditions (14).

3. Steady-state solution

Governing equations 10–13, subject to boundary conditions (14) admit a basic state solution of the form

$$S_s = \widetilde{S}(z) - S_x x - S_y y,\quad \theta_s = \widetilde{\theta}(z) - R_x x - R_y y,$$
$$u_s = u(z),\quad v_s = v(z),\quad w_s = 0,\quad P_s = P(x, y, z), \tag{15}$$

where

$$u = -\frac{\partial P}{\partial x},\quad v = -\frac{\partial P}{\partial y},$$
$$0 = -\frac{\partial P}{\partial z} + \left[\frac{1}{Le}\left(\widetilde{S}(z) - S_x x - S_y y\right) + \widetilde{\theta}(z) - R_x x - R_y y +\right](1 + \gamma z),$$
$$\frac{1}{Le}D^2\widetilde{S} = -uS_x - vS_y,\quad D^2\widetilde{\theta} = -uR_x - vR_y. \tag{16}$$