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Review

Galerkin method for solving combined radiative and conductive heat transfer

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ABSTRACT

This article deals with a numerical solution for combined radiation and conduction heat transfer in a gray absorbing and emitting medium applied to a two-dimensional domain using triangular meshes. The radiative transfer equation was solved using the high order Discontinuous Galerkin method with an upwind numerical flux. The energy equation was discretized using a high order finite element method. Stability and error analysis were performed for the Discontinuous Galerkin method to solve radiative transfer equation. A new algorithm to solve the nonlinear radiative–conductive heat transfer systems was introduced and different types of boundary conditions were considered in numerical simulations. The proposed technique's high performance levels in terms of accuracy and stability are discussed in this paper with numerical examples given.

1. Introduction

The heat transfer models in participating media for scientific purpose or engineering applications such as high temperature processing or thermal insulation involve coupled radiation and conduction heat equations [1–6]. The most fundamental difficulties for the simulation of radiative and conductive heat transfer problems are the nonlinear coupling terms and evolutive boundary conditions. Historically, many simplified models have been used to carry out numerical simulations of radiative transfer phenomena. Generally, two classes of numerical methods have been applied to attempt to solve radiative heat transfer problems namely the Monte Carlo method [7,8] and the deterministic methods [9–11].

In this study, we chose the Discontinuous Galerkin (DG) method, a deterministic method, to solve the radiative transfer equation (RTE). The DG method is known to be locally conservative, stable and high-order accurate. It can also easily handle complex geometries, irregular meshes with hanging nodes and allows for different degrees of approximation for different elements [12,13]. The DG method is known to be a particularly powerful numerical scheme for the simulation of hyperbolic transport problems. The nodal DG method can be easily combined with the well known finite element methods (FEM) or finite volume methods (FVM) to couple radiative and conductive heat transfer. Spatial DG techniques applied to discrete-ordinates radiation transport have been pioneered by Reed and Hill [14] and Lesaint and Raviart [15].

In Refs. [16], the DG method was extended to the solution of radiative heat transfer problems in absorbing, emitting and scattering media. In Ref. [16] the authors studies a centered numerical flux and implemented a parallel computing algorithm based around the localized DG formulation. In Ref. [17] a 3-d model was reduced to 2-d using symmetry and the DG formulation. In Refs. [18,19], the authors added a stabilization term to penalize the jump of the solution across interior faces of the triangulation and to obtain physical numerical solutions in practical applications. For the transient state, many authors have considered radiation and conduction coupled problem, see Refs. [20–22]. Mishra et al. [22], applied the lattice Boltzmann method to solve the energy equation of a transient conduction–radiation problem in a 2-d rectangular enclosure and the collapsed–dimension method was implemented

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Nomenclature

c_p	specific heat capacity, J kg^{-1} K^{-1}
$\mathcal{C}^0(\Omega)$	continuous functions which vanish at infinity
$\mathcal{D}$	unit disk of $\mathbf{R}^2$: $\mathcal{D} = \{\beta \in \mathbf{R}^2 : \lvert\beta\rvert \leqslant 1\}$
$G^\star$	incident radiation intensity, W m^{-2}
g	dimensionless radiative boundary condition
$I^\star$	radiation intensity, W m^{-2} sr^{-1}
I_b	blackbody intensity, Wm^2 sr^{-1}
h_c	convective exchange coefficient at the boundary, W m^{-2} K^{-1}
k_c	thermal conductivity of the medium, W m^{-1} K^{-1}
L	thickness of the medium, m
n	refractive index
$\mathbf{n}$	outward unit normal to the boundary
N_s	conduction radiation parameter
Bi	Biot number
Q_{rad}	radiative source term, W m^{-2}
$s^\star$	path length, m
s	dimensionless path length
$S^\star_{rad}$	radiative source term, W m^{-3}
t	time, s
$\mathcal{T}_h$	triangulation of Ω
T^*, T^*_0	temperature, initial temperature in the medium, K
T, T_0	dimensionless temperature, initial dimensionless temperature in the medium
T_{ref}	reference temperature, K
Greek symbols	
Ω	dimensionless bounded domain in $\mathbf{R}^2$
$\Omega^\star$	bounded domain in $\mathbf{R}^2$, m
∈	wall emission factor
κ	absorption coefficient, m^{-1}
σ_B	Stefan–Boltzmann constant
ρ	diffuse reflectivity or density, kg m^{-3}
β	direction of propagation of radiation
$\Delta t, \Delta\xi$	time step s, dimensionless step time
Γ	boundary of control volume
ξ	dimensionless time
$\theta = \frac{\kappa^2 L^2}{N_s}$	dimensionless constant
Subscripts	
l	angular direction
h	step of the mesh

to solve the RTE. Asllanaj et al. [20], simulated transient heat transfer by radiation and conduction in two-dimensional complex shaped domains with structured and unstructured triangular meshes working with an absorbing, emitting and non-scattering gray medium. To solve the RTE, these authors applied a modified FVM based on a cell vertex scheme combined with a modified exponential scheme where temperature was approximated by linear interpolation using nodal values. The PHAML (Parallel Hierarchical Adaptive Multi Level) code was used to solve the heat conduction equation in space, with low or high order finite elements.

The objective of the work covered in this paper was to solve the nonlinear system which describes the heat transfer in a gray absorbing and emitting medium. We introduced a high order DG method based on an upwind stable numerical flux to solve the radiative heat transfer problem and a high order finite element method to solve the energy equation. Indeed, the high order Galerkin method ensures a more accurate approach with a computational time gain. We can also generalize this method to solve combined radiation conduction heat transfer in irregular geometries. An adequate norm was introduced to prove the stability of DG method for solving RTE and we shall also give an adapted error estimate which we found to be accurate in our simulations. A new numerical algorithm to solve the considered nonlinear-coupled radiative–conductive problem is given in this paper along with numerical results for high order scheme. We investigated a new type of boundary conditions and obtained new numerical results for cases not covered in previous publications. Finally we shall show that numerical stability was verified in our simulations.

The outline of this paper is as follows. In the next section, the radiative–conductive heat transfer in the absorbing emitting medium will be briefly described and the dimensionless form is introduced. The radiative boundary condition and its corresponding mixed thermal boundary conditions will be presented. Discretization of the RTE and energy equation by the discrete DG method and the finite element method respectively, are discussed in detail in Section 3 and also established the stability of the DG method with the adequate norm. Finally, in Section 4, we shall investigate the coupled heat transfer problem and give results showing the effects of the conduction–radiation parameter and wall emissivity on radiative and conductive heat fluxes.

2. Problem statement

In this section, the radiative–conductive heat transfer system of partial differential equations in a two dimensional gray absorbing and emitting medium is introduced. We denote

$$\partial\Omega^\star_- = \{(s^\star, \beta) \in \partial\Omega^\star \times \mathcal{D} \text{ such that } \beta \cdot \mathbf{n} < 0\}. \tag{1}$$

The unknown of the RTE is the radiation intensity denoted $I^\star(t,s^\star,\beta)$ given at time t, position $s^\star$ and in the direction β. The unknown of the energy equation is the temperature $T^\star(t,s^\star)$ at time t and position $s^\star$. The RTE is given by (see, [20])

$$\beta \cdot \nabla I^\star(t,s^\star,\beta) + \kappa I^\star(t,s^\star,\beta) = \kappa n^2 I_b(T^\star(t,s^\star)), \quad \text{for } (t,s^\star,\beta) \in [0,\tau^\star] \times \Omega^\star \times \mathcal{D}, \tag{2a}$$

$$I^\star(t,s^\star,\beta) = g^\star(t,s^\star,\beta), \quad \text{for } (t,s^\star,\beta) \in [0,\tau^\star] \times \partial\Omega^\star_-, \tag{2b}$$

where κ is the absorption coefficient of the medium and n is the refractive index. In this work, the refractive index and the absorption coefficient are assumed to be equal to one, $n = 1$, $\kappa = 1\ m^{-1}$. $I_b(T^\star)$ is the radiation intensity of the blackbody with, $T^\star$, the temperature of the medium:

$$I_b(T^\star) = \frac{\sigma_B}{\pi} T^{\star 4}, \tag{3}$$

where $\sigma_B = 5.6698 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$ is the Stefan–Boltzmann constant. The radiative boundary condition $g^\star(t,s^\star,\beta)$ takes into account of two quantities, the emitted radiation intensity and the incoming radiation. For an opaque wall with specular reflection, we have