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Regularization opportunities for the diffuse optical tomography problem

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ABSTRACT

In optical tomography, the radiative properties of the medium under investigation are estimated from information contained in measurements provided by a set of light sources and sensors located on the frontier of the probed medium. Such a non-linear ill-posed inverse problem is usually solved through optimization with the help of gradient-type methods. Since it is well known that such inverse problems are ill-posed, regularization is required. This paper compares three distinct regularization strategies for two different optimization algorithms, namely the damped Gauss–Newton and BFGS algorithms, for the two-dimensional diffuse approximation to the radiative transfer equation in the frequency domain. More specifically, the mesh-based regularization is combined with the Tikhonov regularization in the damped Gauss–Newton algorithm. For the BFGS algorithm, the mesh-based regularization is combined with the Sobolev gradients method. Moreover, a space-dependent Sobolev gradients method is proposed for the first time. The performance of the proposed algorithms are compared by utilizing synthetic data. The deviation factor and correlation coefficient are used to quantitatively compare the final reconstructions. Also, three levels of noise are considered to characterize the behaviour of the proposed methods against measurement noises. Numerical results indicate that the BFGS algorithm outperforms the damped Gauss–Newton in many aspects.

1. Introduction

In optical tomography, the volumetric radiative properties of the medium under investigation are estimated from information contained in measurements provided by a set of light sources and sensors located on the frontier of the probed medium [1,2]. The main applications of the optical tomography concern the biomedical domain based on the fact that knowledge of the radiative properties provides information on the physiopathological condition of biological tissues. These applications include the imaging of finger joints [3], breast imaging [4,5], functional imaging of brain activity [6,7] or small-animal imaging [8] for instance.

In order to looking deep into tissue, near-infrared light from about 650 to 900 nm is delivered through optical fibres because the absorption of light is relatively low in this spectral range [9]. However, the light is highly scattered at these wavelengths so that no direct methods can be employed to solve this problem. As a result, the reconstruction of the radiative property maps is usually performed through the solution of an inverse problem consisting in minimizing a cost function which depends on the discrepancy, in a least-square sense, between the measurements and associated predictions [1]. This method thus relies on a forward model that provides predictions assuming the distribution of radiative properties inside the medium is known.

The commonly accepted light propagation model in participating media is the radiative transfer equation [10]. This latter equation is integro-differential and thus heavy computations are needed to get accurate solutions. The computation may become highly CPU-time consuming when dealing with inverse problems for which solutions require large numbers of iterations. Alternatively, if the absorption coefficient of the medium is negligible compared to its scattering coefficient and if the medium does not contain void-like regions (i.e. regions in which the absorption and scattering are very low), then the diffuse approximation to the radiative transfer equation provides a good approximation for

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describing light propagation in the medium [1,9]. In this case one talks about the diffuse optical tomography problem and the properties of interest are reduced scattering and absorption coefficients, denoted $\sigma(\mathbf{x})$ and $\kappa(\mathbf{x})$, where $\mathbf{x}$ is the space variable. The diffuse approximation in the frequency domain is considered in this paper. Note that the nonuniqueness of the simultaneous reconstruction of absorption and reduced scattering coefficients in steady-state diffuse optical tomography has been demonstrated in Ref. [11].

Many works have been undertaken over the past two decades to solve this non-linear ill-posed inverse problem, including the non-linear conjugate-gradient method [12], Gauss–Newton based methods [13–16] associated with a wavelet multi-scale method in Refs. [17,18], shape-based reconstruction method [19,20] or, in a Bayesian framework, the approximation error method [21,22]. While the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm was successfully used to solve the optical tomography problem based on the radiative transfer equation in Refs. [23–26], it is rather surprising to note that its use was not considered, to the best knowledge of the authors, for the solution of the diffuse optical tomography problem (excepted in the very recent paper [27] dedicated to the application of a wavelet multi-scale method associated with the BFGS method).

This paper is dedicated to a comparison between the BFGS algorithm and the damped Gauss–Newton algorithm when solving for the diffuse optical tomography problem. It is well known that such an inverse problem is ill-posed, thus regularization is mandatory. In this paper, three regularization tools are developed: i) mesh-based parameterization of the control variables [28]; ii) Tikhonov regularization; and iii) Sobolev gradients method [29]. More specifically, the mesh-based regularization is combined with the Tikhonov regularization in the damped Gauss–Newton algorithm. For the BFGS algorithm, the mesh-based regularization is combined with the Sobolev gradients method. In addition, a comparison between two strategies is proposed, namely a gradient scaling and an adimensionalization of the control variables, to deal with the different orders of magnitude between the two distributions to be retrieved. In order to quantitatively compared the proposed methods, deviation factor and correlation coefficient will be used in the numerical part to quantify the accuracy of the reconstructions. Moreover, in order to fully characterize the behaviour of the proposed methods against measurement noises, three levels of noise will be considered.

Before presenting the work carried out in the paper, it should be noted that the efficiency of the damped Gauss–Newton algorithm to solve the diffuse optical tomography problem was shown in Ref. [14]. Also, it is worth mentioning that the mesh-based regularization has been studied in Ref. [30] when solving for the diffuse optical tomography problem with the Gauss–Newton method associated with the Tikhonov regularization.

The novelties of the paper can be summarized as follow:

- The quasi-independence of the quasi-optimal Tikhonov parameter with the dimension of the control-space is shown and illustrated in the under-determined case (i.e. when the dimension of the control-space is greater than the number of measurements).
- A BFGS algorithm which combines together the mesh-based regularization and the Sobolev gradients method is designed to solve the diffuse optical tomography problem.
- A space-dependent Sobolev gradients method is developed and successfully applied for the solution of the diffuse optical tomography problem solved by the BFGS algorithm. This regularization tool is totally new, to the best knowledge of the authors, in the field of inversion.

The paper, which constitutes an extension of a previous work [31], is organized as follows: section 2 presents the forward model and its solution by the finite element method. This section also defines the cost function to be minimized. Section 3 brings mathematical tools on: i) strategies to deal with the different orders of magnitude between the two distributions to be retrieved; ii) directional derivatives of first and second order of some functions; and iii) finite element parameterization of the control variables. Section 4 presents in detail the optimization algorithms. Section 5 provides numerical results on a two-dimensional bounded domain to compare the different proposed algorithms. Finally, a detailed derivation of the cost function gradient with the adjoint-state method and through a continuous Lagrangian formulation is provided in Appendix A.

As final comment about the current paper, it is worth mentioning that other types of penalization than the one of Tikhonov have been used when solving for the diffuse optical tomography problem with the Gauss–Newton method such as non-convex and nonquadratic penalizations [32,33]. The use of such penalization terms is out of the scope of the paper.

2. Problem statement

2.1. Forward model

The most commonly used forward model in optical tomography is the diffuse approximation to the radiative transfer equation considered in the frequency domain. This is a simple equation governing the evolution of the photon density within the medium $\mathscr{D}$, say φ. The diffuse approximation model is written as [1,16,22]:

$$
\begin{aligned}
&-\nabla\cdot[D(\mathbf{x})\nabla\varphi_l(\mathbf{x})] + \left[\kappa(\mathbf{x}) + \frac{2\pi \mathrm{i}\nu}{c}\right]\varphi_l(\mathbf{x}) = q_l\delta(\mathbf{x}-\mathbf{x}_l) \ \ \forall \mathbf{x}\in\mathscr{D} \\
&\varphi_l(\zeta) + \frac{A}{2\gamma_{n_\mathscr{D}}} D(\zeta)\nabla\varphi_l(\zeta)\cdot\mathbf{n} = 0 \quad \forall\zeta\in\partial\mathscr{D}
\end{aligned} \tag{1}
$$

where $\varphi_l : \mathscr{D}\to\mathbb{C}$ is the state variable providing predictions for the l-th pointwise collimated light source, $\mathbf{x} = (x_1, \ldots, x_{n_\mathscr{D}})$ stands for the space variable, $n_\mathscr{D}$ is the dimension of $\mathscr{D}$, $D(\mathbf{x}) = [n_\mathscr{D}(\kappa(\mathbf{x}) + \sigma(\mathbf{x}))]^{-1}$ is the macroscopic scattering coefficient (expressed in cm^{-1}), where $\kappa(\mathbf{x})$ and $\sigma(\mathbf{x})$ are respectively the absorption and reduced scattering coefficients (both expressed in cm^{-1}), q_l is the strength of the l-th source whose modulation frequency equals ν, δ is the Dirac delta function, $\mathbf{x}_l$ is located at the distance $1/\sigma$ below the site of the l-th source which is located on the frontier of $\mathscr{D}$, $\partial\mathscr{D}$, i is the imaginary unit, c is the speed of light in the medium $\mathscr{D}$, $\mathbf{n}$ is the outer unit normal vector at $\partial\mathscr{D}$, $\gamma_{n_\mathscr{D}}$ is a dimension-dependent constant ($\gamma_2 = 1/\pi$, $\gamma_3 = 1/4$) [22] and A is a parameter that can be derived from the Fresnel laws if specular reflection is considered [16], or from experimental set-ups [34].

This diffuse approximation to the radiative transfer equation is used when the specific intensity is assumed to be quasi-isotropic everywhere in the medium. A detailed description of the diffuse approximation to the general radiative transfer equation is given in Ref. [1]. It is well accepted that the diffuse approximation model is a reasonably good approximation of the radiative transfer equation as soon as the medium under consideration is highly scattering and satisfies $0 \ll \kappa \ll \sigma$.

The partial differential eq. (1) is solved by the finite element method. To do so, let us define:

$$
\widehat{L}^2(\mathscr{D}) = \left\{ f : \mathscr{D}\to\mathbb{C} \ \text{such that} \ \int_\mathscr{D} \left|f(\mathbf{x})\right|^2 \, \mathrm{d}\mathbf{x} < +\infty \right\} \tag{2}
$$