Applied Energy 228 (2018) 324–338

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

On establishing an analytical power capture limit for self-reacting point absorber wave energy converters based on dynamic response

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HIGHLIGHTS

- Proposed a new analytical power capture bound for SRPAs based on dynamic response.
- Proposed a new constraint equation relating the optimal float to spar impedance.
- Numerically demonstrated how the bound can be approached.
- Introduced inerters to wave energy community to implement optimal geometry control.
- Proposed a new condition for null power capture of SRPAs based on geometry control.

ARTICLE INFO

Keywords:
Self-reacting point absorbers
Power capture limit
Geometry control
Mechanical circuits
WEC canonical form
Impedance matching
Thévenin's theorem
Inerter technology

ABSTRACT

To be a competitive supply of renewable energy, the power capture performance of ocean wave energy converters must improve. This requires that wave energy converter designers identify and invest resources to develop devices that exhibit a strong Technology Performance Level early in the development process. We contend that completing this identification process at the conceptual design stage requires a generalized method to establish the power capture upper bound for any given wave energy converter architecture. This upper bound must reflect simultaneous implementation of both optimal geometry control and power take-off force control – components known to be essential to optimizing performance but difficult to envision for complex WEC architectures.

In this work, we develop and demonstrate a procedure, built on the mechanical circuit framework, to identify this upper bound for a self-reacting point absorber with an inertial modulation mechanism performing the geometry control. We illustrate how the analytical procedure generates generic design guidance, required to achieve the bound, without committing to a specific technology. We follow by formally introducing a new technology into the wave energy community, the inerter, capable of implementing the design guidance to enact the required geometry control. Finally, we apply the analytics within a numerical case study of a previously published wave energy converter configuration, and compare the power capture production of that device to one with equivalent hydrodynamics, but with the new geometry control feature set suggested by the new analytical procedure. Our analysis reveals the potential for a ten-fold increase in power capture even under stringent relative displacement constraints.

1. Introduction

Per unit area of ocean surface, the energy density of the conventional wave energy transport is understood to be ten times greater than the equivalent solar energy flux [1]. The annual offshore global wave energy resource is estimated to yield 16,000 TWh of energy, and this abundant availability has sparked further detailed resource forecasting studies [2–6]. However, the path to commercialization of Wave Energy Converter (WEC) devices is challenged by a dispersion of finite resources developing conceptually diverse WEC designs, thus impeding convergence on a single design architecture as witnessed in the wind energy sector [7–10]. To promote design convergence, Weber introduced the Technology Performance Level (TPL) metric [11] and emphasized the importance of assessing TPL early in a development program to identify technologies with a strong predicted performance once in their commercial state [12]. Robust TPL assessments are needed

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https://doi.org/10.1016/j.apenergy.2018.06.099
Received 28 January 2018; Received in revised form 14 May 2018; Accepted 20 June 2018

Nomenclature

Acronyms

PTO	Power Take-Off
SRPA	Self-Reacting Point Absorber
TPL	Technology Performance Level
TRL	Technology Readiness Level
WEC	Wave Energy Converter
COE	Cost of Energy

Parameters

β	transmission ratio
δ	ratio of bounded to unbounded displacement amplitude
ω	excitation angular frequency of the incoming wave elevation time series (rad/s)
$\widehat{F}_{ex1}$, $\widehat{F}_{ex2}$	complex amplitude of hydrodynamic excitation force on float, spar (N)
$\widehat{F}_{FS1}$	complex force amplitude exerted by Force Source 1 (N)
$\widehat{F}_i$	complex force amplitude exerted on the intrinsic mechanical impedance of the WEC (N)
$\widehat{F}_{meff}$	complex reaction force amplitude exerted by the inerter (N)
$\widehat{F}_{PTO}$	complex force amplitude exerted on the PTO (N)
$\widehat{F}_{PTO_{Clamp}}$	complex force amplitude exerted on the PTO when the magnitude of the PTO impedance is infinite (N)
$\widehat{F}_{Zeq1}$, $\widehat{F}_{Zeq2}$	complex force amplitude exerted on the equivalent float, spar (N)
$\widehat{F}_{Zeq1_{free}}$, $\widehat{F}_{Zeq2_{free}}$	complex force amplitude exerted on the equivalent float, spar when the PTO impedance is zero (N)
$\widehat{F}_{Zeq1_{Clamp}}$, $\widehat{F}_{Zeq2_{Clamp}}$	complex force amplitude exerted on the equivalent float, spar when the magnitude of the PTO impedance is infinite (N)
J	moment of inertia of the inerter flywheel (kg·m^2)
k_3	linear spring stiffness coefficient coupled between the spar and reaction mass (N/m)
m_1, m_2, m_3	mass of the float, spar, and reaction mass (kg)
m_{eff}	inertance of the inerter (kg)
$m_{eff_{opt}}$	optimal inertance to maximize power capture (kg)
P_U	average useful power capture per excitation angular frequency (W)
$P_{U_{Max}}$	average useful power capture per excitation angular frequency under complex conjugate PTO force control (W)
$P_{U_{Max(AC)}}$	average useful power capture per excitation angular frequency under amplitude PTO force control (W)
$P_{U_{Max}}\|_{opt}$	average useful power capture per excitation angular frequency under complex conjugate PTO force control and optimal geometry control (W)
P_U^C	stroke limit constrained average useful power capture per excitation angular frequency (W)
$R_{eq2_{\min}}$	resistance of equivalent spar required for null power capture (Ns/m)
$R_{FS1_{opt}}$	optimal resistance of Force Source 1 to achieve the power capture limit (Ns/m)
$\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$	complex velocity amplitude of float, spar, and reaction mass (m/s)
$\hat{u}_{1_{free}}$, $\hat{u}_{2_{free}}$	complex velocity amplitude of float, spar, when the PTO impedance is zero (m/s)
$\hat{u}_{Clamp}$	complex velocity amplitude of float and spar, when the magnitude of the PTO impedance is infinite (m/s)
$\hat{u}_r$	complex relative velocity amplitude between the float and spar (m/s)
$\hat{u}_{r_{Free}}$	complex relative velocity amplitude between the float and spar when the PTO impedance is set to zero (m/s)
$\hat{u}_{r_{opt}}$	complex optimal relative velocity amplitude between the float and spar when the complex conjugate impedance matching conditions are satisfied (m/s)
X_{EM}	reactance of the Equivalent Mass of the spar (Ns/m)
$X_{eq2_{\min}}$	reactance of the equivalent spar required for null power capture (Ns/m)
$X_{FS1_{opt}}$	optimal reactance of Force Source 1 required to achieve the power capture limit (Ns/m)
Z, R, X	generic mechanical impedance, resistance, and reactance (Ns/m)
Z_{A11}, Z_{A22}	hydrodynamic added mass impedance of float, spar (Ns/m)
Z_{B11}, Z_{B22}	hydrodynamic radiation damping impedance of float, spar (Ns/m)
Z_C	coupled radiation impedance between float and spar (Ns/m)
Z_{eq1}, Z_{eq2}	equivalent impedance of float, spar (Ns/m)
$Z_{eq2_{opt}}$	optimal equivalent impedance of the spar required to achieve the power capture limit (Ns/m)
Z_{FS1}, X_{FS1}	impedance, reactance of force source 1 (Ns/m)
Z_i	intrinsic mechanical impedance of the WEC (Ns/m)
Z_{K1}, Z_{K2}	hydrostatic buoyancy stiffness impedance of float, spar (Ns/m)
Z_{m1}, Z_{m2}, Z_{m3}	mass impedance of float, spar, reaction mass (Ns/m)
Z_{meff}	inerter mechanical impedance (Ns/m)
Z_{mH2}	impedance of combined spar mass and spar hydrodynamics (Ns/m)
Z_{PTO}	PTO mechanical impedance (Ns/m)

to focus resources on developing complex WEC innovations with strong predicted power capture that, while challenging to perfect, are essential to long term techno-economic viability.

We contend that the importance of identifying these innovations cannot be understated. At present, for ocean wave energy to be a cost competitive source of renewable energy, the current cost of energy (COE) must be reduced by a factor of two [13]. Such drastic cuts in COE will not come solely from economies of scale in the manufacturing process – disruptive changes in WEC *architectures* (i.e. design topologies) that induce step changes in performance need to be discovered. The search for these new WEC *architectures* requires casting a wide net over the conceptual design space. Thus, a fast, accurate and sufficiently general method to establish the best possible TPL (i.e. the *power capture upper-bound*) of a new WEC *architecture* during the conceptual design stage (e.g. at low Technology Readiness Level) [11] is needed to steer WEC developers to converge towards promising innovations.

The goal of this work is to demonstrate the process of determining this *power capture upper bound* through a case study using the well-known Self-Reacting Point Absorber (SRPA) WEC architecture. In this case study, we will: (1) establish an analytical method to determine a new analytical upper bound for the “hydrodynamic wave power absorption” [12], (2) create generic design guidance in the form of an analytical expression detailing the constraints between hydrodynamic and inertial properties of the WEC that are essential to achieving the upper bound, (3) use this design guidance to propose a new technology, the *inerter*, to implement the predicted power capture improvements, and (4) examine how this generic design guidance can steer the discovery of new technology innovations for wave energy converter design.

1.1. Analytical methods

The typical design process for WECs follows by selecting: (1) a WEC device *class* based on operating principle (e.g. oscillating surging flaps),