

# Drastic bandwidth enhancement of bistable energy harvesters: Study of subharmonic behaviors and their stability robustness



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## HIGHLIGHTS

- Subharmonic behaviors of bistable energy harvesters are analytically studied.
- Stability robustness is introduced to define their sensitivity to disturbances.
- Experimental results confirmed the pertinence of the analytical predictions.
- Exploiting subharmonic behaviors leads to a 180% increase of the bandwidth.

## ARTICLE INFO

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## ABSTRACT

In order to provide a serious alternative to chemical batteries for the energy supply of isolated sensors, bistable generators have been enthusiastically highlighted in recent years for their ability to harvest vibration energy over a wider frequency range compared to linear generators. Nevertheless, these bistable harvesters are generally characterized through a frequency sweep which does not reveal all the steady-state behaviors they can reach and therefore their full energy harvesting potential. Among such behaviors, subharmonic motions are hidden by this classical characterization and therefore had not received a lot of attention. This study proposes an original complete analytical analysis of subharmonic orbits for energy harvesting to predict their contribution to the global bandwidth of bistable generators. In addition, a new criterion, referred as stability robustness, is introduced to estimate the sensitivity of those behaviors to disturbances of different levels, allowing to finely and accurately estimate suitable behaviors for energy harvesting purposes in realistic conditions (behaviors easy to reach and maintain in time). Experimental results conducted with a buckled beam based electromagnetic generator confirm the relevance of this criterion showing good agreement with the analytical predictions. Subharmonic behaviors finally appear, both theoretically and experimentally, to be of significant interest, as exploiting them leads to a 180% increase of the global operating frequency range of the considered bistable energy harvester, for which more than 100  $\mu$ W are generated on a 70 Hz bandwidth at 0.5 g.

## 1. Introduction

The race for energy harvesting in recent years is undoubtedly linked to the multiplication of stand-alone, left behind, wireless devices asking for more energetic autonomy that cannot be brought by primary batteries. More particularly, vibrations have been highlighted as an interesting indoor/confined environment energy through the introduction of a wide variety of inertial harvesters composed of a spring-mass system coupled with electromagnetic or piezoelectric transducers for instance [1]. Easy to implement on the vibrating source, those harvesters can improve the wireless device compactness (no need to store

and embed their entire lifetime energy) and reduce the maintenance costs (no need to replace empty batteries). The frequency bandwidth however remains one of the main limiting factors of inertial energy harvesters and their adaptability to non-constant or random vibrations is still a challenging issue.

Linear harvesters are indeed used to amplify ambient vibrations matching their natural frequency as studied by Williams et al. [2] but their performance dramatically drops in the close neighborhood of this particular frequency. Erturk et al. [3] first reported that nonlinear bistable harvesters show promising results with enhanced frequency bandwidth compared to linear harvesters [4–8]. Moreover, it is

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interesting to note that the classical method to determine the behaviors of bistable harvesters as a function of the ambient frequency does not reveal the entire richness of their spectrum. Classical frequency sweep characterization may indeed miss behaviors as it only consists in smoothly and slowly increasing or decreasing the excitation frequency and recording the associated behaviors. As the nonlinear aspect of bistable harvesters allows different behaviors to coexist at the same frequency, those frequency-sweep obtained behaviors may not be the only existing ones on the full frequency range. Hidden behaviors may then be used to magnify even more the already extended frequency bandwidth of bistable harvesters.

Among them, subharmonic behaviors have already been noticed in mechanical studies [9–12], but without any true exploitation. Arireta et al. [13] and Syta et al. [14] investigated the energy harvested on some subharmonic behaviors but did not evaluate their influence on the global frequency bandwidth of the harvester. However, the authors have already shown, in a purely experimental study, that the use of these behaviors could triple the useful frequency range of the bistable energy harvesters [15].

To confirm the observations made in the previous experimental study, this article first focuses on the analytical investigation of these original subharmonic behaviors, by analytically evaluating their contribution to the bandwidth enhancement of bistable harvesters. To deepen the analytical analysis, a novel stability criterion referred as stability robustness is introduced to estimate the sensitivity of those behaviors to disturbances of different levels. For high stability robustness, the behaviors will be considered suitable for energy harvesting purposes because they are easy to reach and maintain over time thanks to their low sensitivity to disturbances.

Some considerations on the robustness of behaviors of bistable oscillators have been already introduced by Lansbury et al. [16] who analyzed numerically the size of the basin of attraction of these behaviors (i.e., they analyzed the size of the area linked to the behavior under consideration on the numerical diagram showing the steady-state behavior reached depending on the initial conditions of the mass). The bigger the basin of attraction of the behavior, the easier it is to reach, and the more robust it is. Later, Harne et al. [17] also developed a robustness criterion for the most common behavior (first harmonic behavior) of bistable harvesters by simulating the effect of adding white noise on a sinusoidal excitation and checking if the behavior under consideration was maintained. In this paper, the approach proposed is still different: the stability robustness criterion is defined by the minimum amount of kinetic energy needed to destabilize the behavior. This minimum amount of kinetic energy is determined analytically following a mathematical method developed in the framework of differential equations known as bootstrap method [18,19]. Moreover, the novel stability robustness criterion proposed here is calculated analytically contrary to criteria proposed by Lansbury et al. or Harne et al. and therefore requires a lower computing time, while, thanks to its semi-analytical formulation, would permit some optimization considerations.

The stability robustness is calculated for both first harmonic and subharmonic behaviors. While many works assessed the stability robustness issue by underestimating the mechanical quality factor, experimental results confirm that taking into account this stability robustness criteria is mandatory to correctly predict the harmonic and subharmonic behaviors of a realistic bistable harvester.

## 2. Problem statement

### 2.1. Mathematical model

Fig. 1 presents a common bistable oscillator configuration obtained through buckling (other configurations can be found in the literature). The frame is submitted to the ambient vibration and the mass position is defined with respect to this frame.

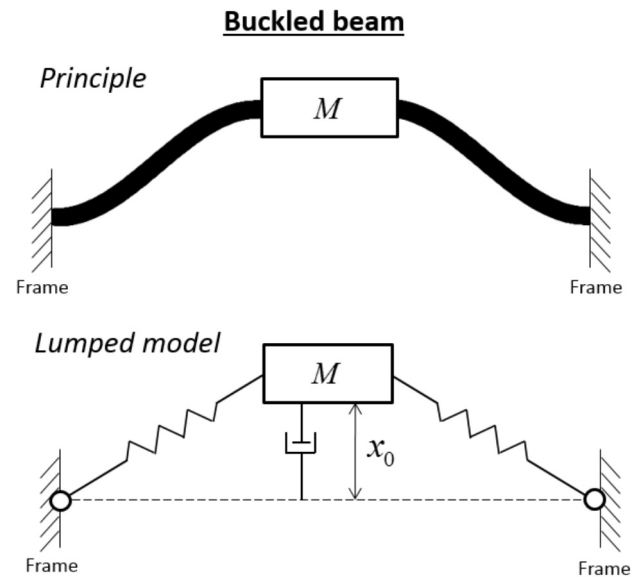


Fig. 1. Principle and lumped model of a common bistable oscillator.

The vibration energy harvester considered in this study is a bistable harvester composed of a generic bistable oscillator coupled with an electromagnetic transducer whose coil is linked to a load resistance for energy harvesting purposes. The electromagnetic transducer converts the kinetic energy of the mass into electrical energy which then dissipates in the load resistance by Joule's effect. The study focuses on this dissipated energy which represents the total energy converted by the bistable harvester. The mathematical model used to describe the bistable harvester is a Duffing-type mechanical equation coupled with the electromagnetic equation:

$$\begin{cases} \ddot{x} + \frac{\omega_0^2}{2} \left( \frac{x^2}{x_0^2} - 1 \right) x + \frac{\omega_0}{Q} \dot{x} + \frac{\beta}{M} I = -A \cos(\omega t) \\ RI = \beta \dot{x} - r_L I - L_0 \dot{I} \end{cases} \quad (1)$$

With  $M$  the mobile mass,  $x$  its relative position with respect to the frame and  $\pm x_0$  its relative stable position.  $\omega_0$  is the natural angular frequency and  $Q$  the mechanical quality factor of the equivalent linear oscillator obtained when the mass oscillates near one of the stable positions ( $x = \pm x_0 + \Delta x$  with  $\Delta x \ll |x_0|$ ) as defined by Liu et al. [20].  $\omega$  and  $A$  are angular frequency and the ambient acceleration.  $\beta$  is the electromagnetic transducer equivalent force factor as defined by Arroyo et al. [21] and  $r_L$  and  $L_0$  are the coil internal resistance and inductance, respectively.  $R$  is the load resistance connected to the coil and  $I$  the current circulating through it. The values of the parameters used in this paper for simulation and analytic resolution correspond to the prototype presented in the experimental part and are listed in Table 1.

The coil impedance is  $\sqrt{r_L^2 + (\omega L_0)^2}$ . For the frequency band under

Table 1

Parameter values of the bistable harvester prototype corresponding to the experimental prototype and used for numerical and analytical analyses.

Parameter	Symbol	Value	Unit
Stable positions	$\pm x_0$	$\pm 0.29$	mm
Inertial mass	$M$	30	g
Natural angular frequency	$\omega_0$	229	rad s <sup>-1</sup>
Mechanical quality factor	$Q$	113	–
Ambient acceleration magnitude	$A$	5	m s <sup>-2</sup>
Ambient acceleration angular frequency	$\omega$	20–200	Hz
Electromagnetic force factor	$\beta$	0.5	N.A <sup>-1</sup>
Coil internal resistance	$r_L$	18	$\Omega$
Coil inductance	$L_0$	5	mH
Load resistance	$R$	18	$\Omega$

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