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Transition between natural laminar and turbulent natural convection along a vertical isotherm wall

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ABSTRACT

This manuscript presents a theoretical study on the transition from laminar to turbulent natural convection flow of gas ($Pr \approx 1$) along a vertical isothermal plate. The analysis of the transition to unstable flows is based on an approximate solution, deduced from the integral formulation of the boundary layer equations for $Pr \approx 1$ (i.e. thickness of the momentum boundary layer is near to that of the thermal boundary layer), and on the Rayleigh equation. An approximate disturbance profile is then calculated. It allows the determination of the parameter $k\delta$ (product of the wave number by the boundary layer thickness) corresponding to retrograde, neutral and progressive wave modes. The Figures 7 to 13 and the Annexes A_1 to A_5 of the ref. [4], edited in 1998 by Elsevier Science Ltd, give to the reader all the necessary information.

The approximate velocity and temperatures profiles used are as follows:

$$\frac{d}{dx}\int_0^\delta u^2 dy = g\beta\int_0^\delta \vartheta dy - \nu\left(\frac{du}{dy}\right)_p \tag{1}$$

$$\frac{d}{dx}\int_0^\delta u\theta\, dy = -\alpha\left(\frac{d\vartheta}{dy}\right)_p \tag{2}$$

1. Introduction

There is a great similarity between continuous heat exchanges, by forced convection and natural convection, if the REYNOLDS number is replaced by the GRASHOF number. This results in a false idea, according to which the transition between natural convection laminar and turbulent flows should itself also depend solely on the GRASHOF and PRANDTL numbers.

When nuclear energy requirements were examined in the 60's, this idea was found to be erroneous. This discovery led to experimental work [1] [2] [3] initially in an atmospheric atmosphere and then in a pressured gas atmosphere of CO_2 or N_2 with $1 < P < 30$ bars.

Older in-depth experimental studies had enabled a precise definition of the transition criteria [3] under atmospheric conditions and with a wall height of 3.20 m.

Existing theoretical studies do not explain the early onset to the transition to turbulence; see for example [8]. However, in 1925 and 1927, two authors [5] and [6] demonstrated the existence of low frequency oscillations occurring in a confined atmosphere, likely to provoke instabilities in the boundary layer through coupling (internal gravity waves oscillations known as BRUNT VAÏSÄLÄ (see Ref. [10]).

This new way was successfully explored in 1998 [4], using the experimental work described in Ref. [1], and based on a scale analysis, as indicated by BEJAN A. in Ref. [12].

The theoretical analysis which follows, confirms the validity of the new approach, which establishes the possibility of coupling between oscillations inherent in the boundary layer and low frequency oscillations which may be present outside it.

2. New theoretical approach

The starting point is the simplified solution for laminar flow, established by SQUIRE and described by E. R. G. ECKERT, Robert M.

[1] « l'auteur n'est affilié a aucun institut ou autre partenaire académique ».

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Notations

A, B, C, D	coefficients designed to indicate the velocity profile of the disturbance [m s^{-1}]
Cp	specific heat at constant pressure [J kg^{-1} K^{-1}]
f	frequency [Hz]
f_{BV}	Brunt Väisälä frequency
h	heat transfer coefficient [W m^{-2} K^{-1}}
k	wave number [m^{-1}]
T_p	temperature of the plane [K]
T_δ	temperature outside the boundary layer [K]
U_1 U_{max} U_{moy}	velocities used to characterize the velocity profile of the boundary layer in steady established regime [m s^{-1}]
u v	components of the velocity of the boundary layer [m s^{-1}]
V	velocity profile of the disturbance [m s^{-1}]
x, y	coordinates [m]
Y=	y/δ

Dimensionless Numbers: See Ref. [14] G. I. BARENBLATT "Scaling" for the dimensional analysis and the physical similarity.

Gr	GRASHOF number $g\beta\,(T_w-T_\delta)\,x^3/\nu^2$
K_1, K_2	Numbers defined from the relations (24) and (28)
Nu	NUSSELT number $h\,x_{tr}/\lambda$
Pr	PRANDTL number μ Cp/λ

Greek characters

α	thermal diffusivity [m^2 s^{-1}]
β	thermal expansion coefficient [K^{-1}]
δ	boundary layer thickness [m]
λ	thermal conductivity [w s^{-1} m^{-1} K^{-1}]
Λ	wave length [m]
μ	dynamic viscosity [kg m^{-1}s^{-1}]
ϕ	surface flux [W m^{-2}]
ρ	density [Kg m^{-3}]
$\nu=$	(μ/ρ) kinematic viscosity [m^{-2} s^{-1}]
$\theta=$	($T_p - T$) [K]
$\psi=$	stream function
$\omega=$	pulsation [2πf t^{-1}]

DRAKE Jr [8]. This approach produces an approximation of the laminar boundary layer profile using an integral method (Fig. 1).

$$u = U_1\frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2 \tag{3}$$

$$\theta = \theta_p\left(1-\frac{y}{\delta}\right)^2 \tag{4}$$

The integration of the equations which precede, with the approximate velocity and temperature profiles, gives the following result:

$$U_1 = C_1 x^{1/2} \tag{5}$$

$$\delta = C_2 x^{1/4} \tag{6}$$

$$C_1 = 5.17\nu\left(\frac{20}{21}+\frac{\nu}{\alpha}\right)^{-1/2}\left(\frac{g\beta\vartheta_p}{\nu^2}\right)^{-1/2} \tag{7}$$

$$C_2 = 3.93\left(\frac{20}{21}+\frac{\nu}{\alpha}\right)^{1/4}\left(\frac{\nu}{\alpha}\right)^{-1/2} \tag{8}$$

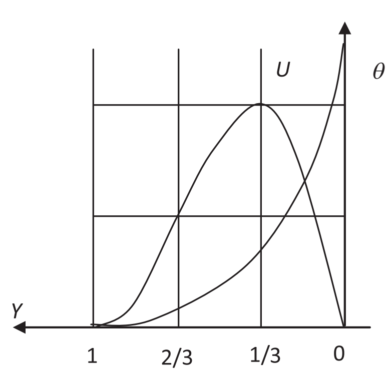

Fig. 1. Approximate velocity and temperatures profiles used are as follows.

$$U_{max} = \frac{4}{27}U_1 = 0.766\nu\left(\frac{20}{21}+P_r\right)^{-1/2} Gr_x^{1/2}\frac{1}{x} \tag{9}$$

$$\frac{\delta}{x} = 3.93Pr^{-1/2}\left(\frac{20}{21}+Pr\right)^{1/4}(Gr_x)^{-1/4} \tag{10}$$

This approximate description of the velocity and thickness of the natural convection laminar layer constitutes the basis of the new theoretical approach to the stability of natural convection laminar flow.

To this effect, consideration of the dynamics equations and the energy equation as described in Ref. [9] on page 427, leads to an Orr–Sommerfeld equation.

Lord Rayleigh noted that laminar flow whose velocity profiles have an inflexion point tend strongly towards instability, the present case demonstrates this.

Although the presence of a point of inflexion is precisely due to the viscosity, in what follows we shall ignore the influence of this viscosity during the phase of transition to unstable flows, in order to arrive at the Rayleigh equation, (Orr–Sommerfeld equation in which the viscous dissipation is ignored), see for example ref. [9] page.431.

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\nabla^2\psi + \left(-\frac{d^2\overline{u}}{dy^2}\right)\frac{\partial\psi}{\partial x} = 0 \tag{11}$$

BEJAN A. and CUNNINGTON G. [11] also used part of this equation in order to develop their theoretical study.

A solution to this equation is sought in the form:

$$\psi(x,y,t) = V(Y)e^{i(kx-\omega t)} \tag{12}$$

$$\text{with } Y = y/\delta \tag{13}$$

In order to solve Rayleigh's equation, BEJAN and CUNNINGTON use a simplified method, conceived initially by Lord Rayleigh in the years 1879 and 1880, and then returned to years later by Sir Horace LAMB ref. [7] p 671.

In this approach, the velocity profile of the boundary layer is simplified to the extreme and is represented by simple line segments. It is an angular profile which includes points of discontinuity, hence the requirement to introduce additional relations in