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Stability of a porous Benard-Brinkman layer in local thermal non-equilibrium with Cattaneo effects in solid

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ABSTRACT

The stability of a porous Benard layer of Brinkman fluid under local thermal non-equilibrium conditions and obeying the Cattaneo flux law in the solid is studied. The preference for stationary and overstable linear motions is examined in the whole parameter space of the problem. It is shown that overstability occupies a region within a branch of a rectangular hyperbola-like curve in the first quadrant of the $(\hat{\tau}, H)$ plane, where $\hat{\tau}, H$ are the Straughan and thermal inter-phase interaction parameters, respectively, so that overstability persists even when $H \to \infty$. The Brinkman effect tends to stabilize the layer but enhances the region of preference of overstability in the parameter space. The influence of the other parameters on the critical mode is also identified. The nonlinear development of the amplitude *A* of the linear wave motions of both stationary and overstable modes is found to be governed by a first order evolution equation. The analysis of the evolution equation shows that the layer can exhibit supercritical instability for both stationary and overstable modes and subcritical instability and stability for the stationary mode depending on the relative magnitudes of the parameters of the problem. The Brinkman effect is found to reduce the amplitude of the supercritical instability while the porosity modified ratio of thermal conductivities and the ratio of thermal diffusivities tend to promote subcritical instabilities. The implication of these results on the nonlinear global stability of the model is discussed.

1. Introduction

The flow of fluids in porous media under local thermal non-equilibrium conditions has recently received considerable attention because of its relevance to many applications ranging from modelling of underground CO_2 sequestration [1] to tube refrigerators [2] to nanofluid flows [3] and many other applications. The reader is referred to the introduction of the book by Straughan [4] for more types of applications and to the study by Virto et al. [5] which gives several causes of local thermal non-equilibrium.

The state of local thermal non-equilibrium is defined as the state in which the solid has a temperature, T_s, different from that, T_f, of the fluid. This demands an energy equation for the solid as well as one for the fluid. One direct approach is to use an equation for the solid that is similar to that for the fluid but with different medium properties [6–13]. Another approach is to use Cattaneo effects on the solid [14]. Cattaneo, realizing that the classical relationship between the heat flux and the temperature gradient leading to the heat equation has a solution that has an infinite wave speed, proposed a modification of the relationship between the heat flux and temperature to produce a solution that possesses a finite wave speed [4, p.11]. The finite speed introduced by the Cattaneo effect is now known as the second sound, while the classical diffusive wave is referred to as the first sound [14,15].

The problem of convection in a porous Benard layer in local thermal non-equilibrium first introduced by Horton and Rogers [16] and Lapwood [17] has recently received considerable attention [18] because it provides a simple model that allows a detailed analysis of the roles of the different parameters of the problem and hence provides good understanding of the basic properties of porous media in local thermal non-equilibrium. Banu and Rees [6] were the first to study the convective instability of a porous layer in local thermal non-equilibrium. They used the equations stated by Nield and Bejan [18] in which the fluid and solid have separate energy equations governing their different temperatures. Both equations use Fourier heat flux law. The two equations are coupled by a thermal inter-phase interaction parameter, *H*, which is a measure of the heat transfer between solid and fluid. Adopting the steady Darcy equation without Brinkman and Forchheimer effects, they found that the layer in local thermal non-equilibrium can be unstable to stationary convective motions only. The energy

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required for exciting the motions, as measured by the Rayleigh number, depends strongly on the thermal inter-phase interaction parameter, H, with small values of the parameter resembling strong local non-equilibrium while large values of the parameter give results similar to those of the layer in local thermal equilibrium.

Straughan and Franchi [19] were the first to introduce Cattaneo effects to the study of convective instability of a porous Benard layer in local thermal equilibrium. They found that the presence of Cattaneo effects introduces convective instabilities when the top plane is hotter than the lower, in which case the layer is thermally stably stratified, in addition to the classical instability present when the bottom plane is hotter. Recently, Straughan [20] extended that study to a layer in local thermal non-equilibrium and identified a global nonlinear criterion for the stability of the layer using an energy method. The stability problem depends on a number of dimensionless parameters: the ratio of the thermal diffusivity of the fluid to that of the solid, B, the porosity modified ratio of the thermal conductivities of the fluid and solid, γ, the thermal inter-phase interaction parameter, H, the Straughan parameter, $\hat{\tau}$, which is the ratio of the relaxation time to the thermal diffusion time of the fluid, and the Rayleigh number, Ra, which measures the thermal energy provided by the applied adverse temperature gradient across the layer (see Equation (2.16) below). The global condition of stability was found to be independent of the parameters $B, \gamma, H, \hat{\tau}$. Straughan also examined the linear stability of the layer to discover that the Cattaneo effect introduces overstable motions to the layer provided the parameter, H, is not too small. He examined the dependence of the Rayleigh number necessary for instability to find that it always lies above that required for the nonlinear global stability criterion.

The motivation for the present study is threefold. First we investigate further the roles played by the parameters $B, \gamma, H, \hat{\tau}$ (see Equation (2.16)) on the linear stability. The study by Straughan [20] showed that the introduction of second sound by the Cattaneo effect introduces overstable modes, which are preferred provided that the interaction inter-phase parameter, H, and the parameter $\hat{\tau}$ are large enough. However, the functional relationship between H and $\hat{\tau}$ on the boundary between regions of preference of stationary and overstable modes has not been examined. We will identify the boundary between the overstable and stationary modes in the $(\hat{\tau}, H)$ plane and discuss its dependence on the other parameters of the problem in Section 3 below. Furthermore, the study [6] by Banu and Rees showed that when H is very large, the stability of the layer becomes identical to that of a layer in thermal equilibrium. The study [20] did not address the limit of very large H on the stability of the layer in the presence of the Cattaneo effect. We investigate this limit in Section 3 below and obtain asymptotic results for the preferred mode at large values of H.

The second motivation is the inclusion of the Brinkman effect (or effective viscosity) of the fluid to the Cattaneo model. A number of studies have considered the convective instability of a porous medium under the Brinkman effect in the fluid [21,22,23]. However, they all included the inertia term in the Darcy equation and consequently their overstable motions are influenced by the inertial oscillations. Here we exclude the inertia term in the Darcy equation so that overstable motions are due to second sound only. The influence of the Brinkman effect on the stability of the stationary and overstable second sound modes as well as its role in the choice of *preference* of the two modes is investigated. The results are summarized in regime diagrams and discussed in Section 3.

The third motivation is to get some insight into whether the nonlinear global condition for stability derived in Ref. [20] using an energy method is necessary as well as sufficient for stability. The study [20] showed that the energy required, as measured by the Rayleigh number, Ra (see Equation (2.16) below), for the linear stability of the layer exceeds that of the global stability for non-zero values of H. This suggests that subcritical instabilities are likely to occur. Also the condition for nonlinear global stability is independent of the parameters $B, \gamma, \hat{\tau}$, and it is of interest to examine whether the occurrence of subcritical instabilities, if they occur, depends on the specific values of the parameters. We also include in this study the influence of the Brinkman effect on the occurrence of subcritical instabilities.

In Section 4 we use a formal weakly nonlinear analysis to show that the amplitude of the linear theory is governed by a first order evolution equation over long time and distance. A detailed analysis is made of the evolution equation with regard to the possible existence of subcritical and supercritical instabilities. In Section 5, we mention some concluding remarks.

2. Formulation

We consider a horizontal layer of saturated porous medium in local thermal non-equilibrium contained between two stress free horizontal planes a distance,d, apart. The basic equations are the steady Darcy–Brinkman equation, the continuity equation and the Nield–Bejan equations for the temperatures of the fluid and solid when the heat flux of the solid obeys the Cattaneo effect while that of the fluid conforms to the Fourier heat flux law:

$$0 = -\nabla p + \rho_f \mathbf{g} - \frac{\mu_f}{K}\mathbf{u} + \hat{\mu}_f \nabla^2 \mathbf{u}, \tag{2.1}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{2.2}$$

$$(1-\varepsilon_p)(\rho c)_s \frac{\partial T_s}{\partial t} = -(1-\varepsilon_p)\nabla \cdot \mathbf{Q} - h\left(T_s - T_f\right), \tag{2.3}$$

$$\tau_s \frac{\partial \mathbf{Q}}{\partial t} = -\mathbf{Q} - k_s \nabla T_s, \tag{2.4}$$

$$(\rho c)_f \left[\varepsilon_p \frac{\partial T_f}{\partial t} + \left(u \cdot \nabla T_f\right)\right] = \varepsilon_p k_f \nabla^2 T_f + h\left(T_s - T_f\right), \tag{2.5}$$

Here p,$\mathbf{u}$,$\mathbf{Q}$,T,ρ are, respectively, the pressure, velocity, flux in the solid, temperature, density and g,K, $\mu_f, \hat{\mu}_f, \varepsilon_p, c, k, h, \tau_s$ are the acceleration due to gravity, permeability, fluid dynamic viscosity, effective viscosity, porosity, specific heat at constant pressure, thermal conductivity, thermal inter-phase interaction coefficient and the relaxation time of the solid, respectively. We have adopted the Boussinesq approximation here so that the density is assumed constant except when it occurs in conjunction with gravity. Also, we shall always use the subscripts f,s to refer to fluid and solid, respectively.

The neglect of the time dependent term in the Darcy–Brinkman equation filters out the inertial modes of the system so that the propagating waves discussed below are second sound waves.

We wish to examine the stability of a stationary basic state in which the two planes are maintained at constant temperatures with the lower plane possessing a higher temperature, T_L, than that, T_U, of the upper plane. We define a Cartesian system of coordinates in which the origin O lies half-way between the two planes, Oz is directed vertically upwards and Ox,Oy are horizontal. The basic state variables, denoted by a 'tilde', are

$$\begin{gathered}\tilde{\mathbf{u}} = 0, \quad \tilde{T}_f = \tilde{T}_s = -\beta z + T_L, \quad \tilde{\mathbf{Q}} = (0, 0, k_s\beta), \\ \tilde{\rho}_f = \rho_0\left[1 - \alpha_t\left(\tilde{T}_f - T_0\right)\right],\end{gathered} \tag{2.6}$$