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Unsteady free convection from a heated sphere in the presence of internal heat generation or absorption

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ABSTRACT

This paper is concerned with an unsteady, laminar, free convective flow over a heated sphere with the effect of internal heat generation/absorption. The dimensionless governing equations have been solved employing the finite difference method as well as a perturbation method for short time and an asymptotic method for long time. We examine the effects of the physical parameters, such as, the Prandtl number, Pr, and the heat generation/absorption parameter, γ, on the friction factor and heat transfer rate as well as the velocity and temperature profiles. It is observed that when the Prandtl number, Pr, is increased, the friction factor decreases while the heat transfer rate increases. In the presence of internal heat generation, the friction factor increases while the heat transfer rate reduces. The reverse pattern is found with the heat absorption parameter. The momentum and thermal boundary layers become thicker with an increase of the heat generation parameter. A comparison among the numerical solutions, the perturbation solutions for short time and the asymptotic solutions for long time has been presented which provides a good agreement among the solutions.

1. Introduction

Due to the occurrence in many technological problems, the free convective flows and heat transfer from hot particles has attracted much attention of researchers [1]. When a heated body is placed in an unbounded fluid environment, an upward convection of fluid and heat arises resulting from the density differences. This characteristic is found to be limited to a thin boundary layer and to the plume above the body. In this case, the buoyancy force plays an important role in fluid flow and thus most of the studies in this field have focused on the problems with large values of Grashof number, which is suitably defined by buoyancy force. On the other hand, when the temperature difference between the surface of a heated body and its surrounding environment is significant, a strong influence of the temperature dependent heat source or sink in the moving fluid is often seen in several practical problems. For example, this class of phenomena takes place in the underground contaminant transport, enhancement cooling of microprocessors [2,3], cooling of the planets [4], combustion and vaporization of fuel droplets, spray drying, heat transfer from packed beds of spherical bodies [5], hydrology, petroleum geology, geophysics, thermal-insulation engineering, nuclear engineering [6], and so on.

Now a typical problem of the free convection is the free-convective flow from a heated sphere. Potter and Riley [7] investigated the free-convective flow from the surface of a heated sphere at high Grashof number. The behavior of the boundary layer growth near the surface of the heated sphere was observed numerically. They found that the solution fails at the upper stagnation point, and the fluid motion suddenly ensues from the boundary layer to form the plume which develops over the sphere.

In order to know the features of power-law non-Newtonian fluids emerging in several practical applications, Acrivos [8] derived an equation for the local Nusselt number in laminar natural convection. It is applicable to any two-dimensional surface or a surface of revolution about an axis of symmetry and for large values of Prandtl number.

For the case of large Grashof number, Brown and Simpson [9] presented a detailed analysis of the unsteady problem in which a sphere kept at rest in a fluid is impulsively heated as time starts. For this problem, two types of singularity have been identified at the upper pole of the sphere. The first one is from a solution of the unsteady equations after a finite time, and the second from a solution of the steady equations.

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Awang and Riley [10] studied the unsteady free-convective boundary layer flow over the surface of a sphere whose temperature is suddenly raised to a uniform value greater than its surroundings. Even though they concluded that the solution gives a complete understanding of the nature of the free-convection boundary-layer flow over the surface of a heated sphere, it exhibits the singularity as in Ref. [9].

The free-convective flow in the vicinity of the upper pole of a heated sphere at high Grashof number was examined by Amin and Riley [11]. They elaborately discussed the effects of a spatially varying surface temperature and demonstrated that a pressure gradient is generated due to the above consequence that induces the flow to converge on the stagnation point at the upper pole of a sphere. In addition, the solution of the boundary-layer equations experiences a singularity which is an evidence of the eruption of fluid from the surface.

Saito et al. [5] examined the transient laminar natural convection flow over a sphere subjected to a constant heat flux and for high Grashof numbers and a wide range of Prandtl numbers. A numerical method was used for the solution of the governing equations. They found that the thermal boundary layer thickness is smaller than the hydrodynamic boundary layer thickness for any Prandtl number.

Jaluria and Gebhart [12] studied the boundary layer natural convection flow over heated hemispheres. They considered both the upright and inverted hemispheres of diameter 15.2 cm and 30.5 cm. A detailed investigation was carried out in terms of the velocity and temperature fields in regions close to the top of the hemisphere. Amato and Tien [13] examined the free convection heat transfer from heated spheres to water. They performed experiments taking four sphere sizes with nominal diameters of 1, 2, 3 and 4 inch. The temperature and velocity fields around a heated isothermal sphere have been measured using the hot-film anemometry techniques.

Not only the flow pattern but also the heat transfer is influenced by the heat flux around a heated sphere. These effects become more effective when there is a heat source or sink within the fluid. A variety of simple mathematical models is used to predict the effect internal heat generation or absorption within a system. During the model development, heat generation or absorption is generally considered to be constant, space-dependent or temperature dependent. Sparrow and Cess [14] studied the steady stagnation point flow and heat transfer assuming temperature dependent heat absorption. The influence of temperature dependent heat sources on the heat transfer within a porous medium has been investigated by Moalem [15]. The effects of heat generation or absorption on the hydromagnetic convection at a cone and a wedge have been presented by Vajravelu and Nayfeh [16]. Chamkha [17] examined the mixed convection in a channel filled with a porous medium using a linear variation with temperature dependent heat sources or sinks. Crepeau and Clarksean [18] considered a space-dependent exponentially decaying heat generation or absorption model to examine the flow and heat transfer from a vertical plate.

Kurdyumov and Liñán [19] investigated the steady free convection flow due to a point source of heat and heated spheres using the Boussinesq equations. Numerical method has been carried out for a wide range of values of the Prandtl number and the Grashof number. They proposed a correlation expression for the laminar flow Nusselt number ranging from small to large Grashof number.

From the above literature survey, we can conclude that most of the works deal with steady problem and/or the solution terminates at a singularity. In this study, we investigate the unsteady laminar free convective flow over a heated sphere in the presence of internal heat generation/absorption. The set of equations describing the present problem has been solved numerically using the finite difference method for a wide range of the time and the short and long time approximated solutions have also been performed. It is found that these solutions provide good approximations to the numerical solutions. We analyze the effects of the physical parameters, such as, the Prandtl number, Pr, and the heat generation/absorption parameter, γ, on the friction factor and the heat transfer rate as well as the velocity and temperature profiles.

2. Formulation of the model

A two-dimensional, unsteady, laminar, free convective flow over a heated sphere has been considered taking into account the heat generation/absorption. The physical configuration and coordinate system is shown in Fig. 1. In the Boussinesq approximation, that is, where variable fluid properties are ignored except in the buoyancy term, the equations of motion, as given in Refs. [7,10], but with the heat generation/absorption in the fluid are

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u}) + \frac{\partial}{\partial \bar{r}}(\bar{r}\,\bar{v}) = 0 \tag{1}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{r}} = -\frac{1}{\rho_\infty}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\nu}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{u}}{\partial \bar{r}}\right) + \frac{\rho - \rho_\infty}{\rho_\infty} g \sin\left(\frac{\bar{x}}{a}\right) \tag{2}$$

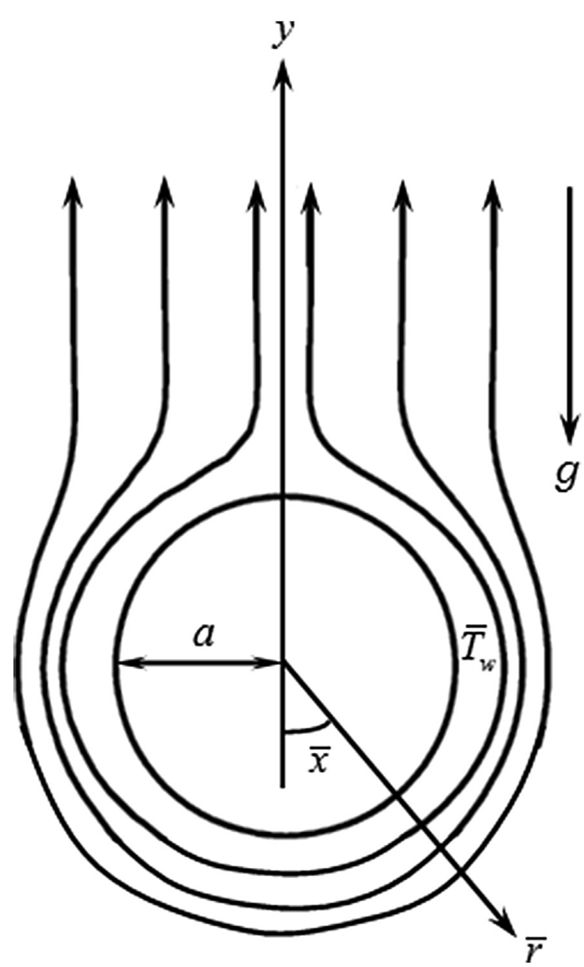

Fig. 1. Physical configuration and coordinate system.