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Linear stability analysis of flow instabilities with a nodalized reduced order model in heated channel

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ABSTRACT

The prime objective of the presented work is to develop a Nodalized Reduced Order Model (NROM) to carry linear stability analysis of flow instabilities in a two-phase flow system. The model is developed by dividing the single phase and two-phase region of a uniformly heated channel into N number of nodes followed by time dependent spatial linear approximations for single phase enthalpy and two-phase quality between the consecutive nodes. Moving boundary scheme has been adopted in the model, where all the node boundaries vary with time due to the variation of boiling boundary inside the heated channel. Using a state space approach, the instability thresholds are delineated by stability maps plotted in parameter planes of phase change number (N_{pch}) and subcooling number (N_{sub}). The prime feature of the present model is that, though the model equations are simpler due to presence of linear–linear approximations for single phase enthalpy and two-phase quality, yet the results are in good agreement with the existing models (Karve [33]; Dokhane [34]) where the model equations run for several pages and experimental data (Solberg [41]). Unlike the existing ROMs, different two-phase friction factor multiplier correlations have been incorporated in the model. The applicability of various two-phase friction factor multipliers and their effects on stability behaviour have been depicted by carrying a comparative study. It is also observed that the Friedel model for friction factor calculations produces the most accurate results with respect to the available experimental data.

1. Introduction

Over the past several years it is evident that two phase flow instabilities are of paramount interest in thermal hydraulic studies [1–6]. Among these instabilities, DWO is a major constituent, which can arise in a boiling system when a constant pressure drop [7,8] is imposed across a heated channel. Predominantly in nuclear reactors, this phenomenon can be triggered both in fuel channels and in steam generators. However, the DWO phenomenon in nuclear reactors is also caused due to the coupled neutronic feedbacks.

The DWO phenomenon can be interpreted as the origin of instability associated with density perturbations in a fluid with delay propagations, causing waves of higher and lower density fluid flowing across a heated channel [5,8]. Broadly speaking, the difference between inlet and exit fluid density in a boiling system triggers delays in the transient distribution of pressure drops across the channel which may lead to self-sustained oscillations that are known as DWOs. In addition to these, the flow instabilities in a natural circulation system are broadly classified into two categories, namely Type-I and Type-II [9–20]. The Type-I instability arises at low power conditions where the steam quality is low. During a low steam quality situation, a slight change in the quality due to any disturbance causes a large variation in void fraction and consequently the driving head changes. As a result the flow fluctuations occur. But at high power conditions, when the void fraction and steam quality is large, the frictional pressure losses increase. Moreover, a large void fraction generates void propagation time delay in the two-phase region of the system. Under these situations, any disturbance in the flow can cause large fluctuations in the fluid density as well as frictional pressure drops. These fluctuations eventually induce the coolant flow rate to oscillate, which is classified as Type-II instability. It is worth noting that the Type-II instability in natural circulation system is classified as Density wave oscillation (DWO). These instabilities are strongly undesirable in aforementioned systems, because continual flow oscillations can cause mechanical vibration to the constituent components thereby affecting the system performance.

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Nomenclature

A^*	cross section area (m^2)
B_t^*	loop width (horizontal section, m)
C_0	void distribution parameter
D_h^*	hydraulic diameter of flow channel (m)
D_{doc}^*	diameter of downcomer (m)
D_r^*	diameter of riser (m)
f	friction factor
Fr	Froude number ($v_0^{*2}/g^* L_{ch}^*$)
g^*	acceleration due to gravity (m/s^2)
h^*	enthalpy (kJ/kg)
H_t^*	downcomer level (m)
J	Jacobian matrix
K_{inlet}	inlet pressure loss coefficient
K_{exit}	exit pressure loss coefficient
L_{ch}^*	total length of flow channel (m)
L_r^*	length of riser (m)
L_h^*	length of standpipe (m)
N_f	friction number ($fL_{ch}^*/2D_h^*$)
N_{pch}	phase change number ($q''^* \Delta\rho^* \xi_h^* L_{ch}^* / A^* \Delta h_{fg}^* v_0^* \rho_g^* \rho_f^*$)
N_{sub}	subcooling number ($(h_{sat}^* - h_{inlet}^*)\Delta\rho^* / \Delta h_{fg}^* \rho_g^*$)
N_s	number of nodes in single phase
N_t	number of nodes in two-phase
q''^*	wall heat flux (W/m^2)
t^*	time (s)
v_0^*	reference velocity (m/s)
v_{inlet}^*	inlet velocity of coolant (m/s)
V_{gj}^*	average drift velocity (m/s)
x	steam quality
z^*	distance along the axis of flow channel (m)
ξ_h^*	heated perimeter (m)
α	void fraction
μ^*	boiling boundary (m)
ρ^*	density (kg/m^3)

Subscripts

1Φ	single phase
2Φ	double phase
doc	downcomer
ex	exit
f	liquid
g	vapour
in	inlet
m	mixture
r	riser

Superscripts

*	dimensional quantity
~	steady state value

Abbreviations

AHWR	advanced heavy water reactor
BWR	boiling water reactor
DFM	drift flux model
DR	decay ratio
DWO	density wave oscillation
HEM	homogeneous equilibrium model
HPNCL	high pressure natural circulation loop
NIST	national institute of standards and technology
NROM	nodalized reduced order model
ODE	ordinary differential equation
PDE	partial differential equation
ROM	reduced order model
SB	stability boundary

Extensive studies have been carried out to analyze the discussed instability phenomenon (DWO). To serve the purpose, two general approaches have been adopted by several researchers namely frequency domain approach and time domain approach. The frequency domain approach [21] is associated with control-theory techniques, where transfer functions are obtained using Laplace transformation of the linearized model of governing equations. The stability of the system (linear stability characteristics with small perturbation) is then determined by the analyses of roots of the obtained transfer functions [9,10,12,13,15,22]. However, it is pointed out that this technique is not very useful for analyzing the nonlinear stability characteristics (for large perturbation) of the system due to the presence of linearized equations. Moreover, in time domain approach, the conservation equations are either analytically integrated [23–26] over the competing regions, or by applying suitable numerical techniques, 1-D analyses [27–29] are carried out. In this type of analysis, the steady state of the system is perturbed and the system evolution is observed with stepwise gradual change in operating parameters. The points where undamped or diverging oscillations are obtained are noted as the stability thresholds. It is also pointed out that using these methods, the nonlinear stability analysis of the system is computationally intensive. Because to gain knowledge of nonlinear stability characteristics, the governing equations need to numerically simulate for wide range of parameters. In contrast, the study of nonlinear dynamics being a theme of interest for last two decades, nonlinear stability analyses of heated channels have become significantly advanced followed by bifurcation analysis. Achard et al. [30] carried out an analytical bifurcation study of DWO with homogeneous equilibrium model (HEM). Rizwan-Uddin and Dorning [31] extended that approach by replacing HEM by drift flux model which results even more complicated nonlinear delay integro-differential equations. In their study they showed the impact of void distribution parameter C_0 and the drift velocity V_{gj} between phases on the sensitivity of stability boundary (SB) in various planes.

In addition to the mentioned techniques, Reduced Order Models (ROMs) are also available where using a state space approach, the linear as well as nonlinear stability characteristics of the system can be determined. Clausse and Lahey [32] developed a simple model using approximations for spatial dependence of the single phase enthalpy and two-phase density. Followed by these developments Karve [33], Dokhane [34] and Dokhane et al. [35] developed models for DWO by introducing quadratic profiles of single phase enthalpy and two-phase quality using HEM as well as drift flux model (DFM). However due to the presence of quadratic profiles, the system equations became highly complicated and even run for several pages. It should be noted that using linear–linear approximations of single-phase enthalpy and two-phase quality; a simpler model compared to the aforementioned models had also been presented by Karve et al. [36], where though the model equations remain simpler, but the results obtained showed significant discrepancy with the experimental data sets [37].

The main objective of the current research is to extend the concept of the model developed by Paul and Singh [38], while keeping a new insight to analyze the system stability characteristics by diving the flow channel into N-number of nodes. To achieve the