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Influence of slip wall effect on a non-isothermal electro-osmotic flow of a viscoelastic fluid

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ABSTRACT

We present an analytical model that predicts the influence of Joule heating on the slip velocity in an electro-osmotic flow (EOF) of viscoelastic fluids. The viscoelasticity of the fluid is taken into account by employing the simplified Phan-Thien and Tanner constitutive model (sPTT). The Joule heating induces temperature gradients along the microchannel making properties non-uniform and hence alters the electric potential and the flow field. In consequence, the slip velocity and the velocity gradient on the microchannel surface are drastically modified in comparison with the case of uniform properties. Using the well-known lubrication theory, the momentum equations together with the energy, Poisson and Ohmic current conservation equations are considerably simplified. The dimensionless mathematical model is solved by using a regular perturbation technique, which is compared against a numerical solution. The results show that using hydrophobic microchannels, for the used values of the parameters involved in this analysis, the volumetric flow rate through microchannels can be massively amplified in about 400%, in comparison with the case of non-slipping surfaces. In addition, by using hydrophobic microchannels, the maximum temperature in the microchannel can be substantially reduced.

1. Introduction

Recently, EOF has found wide applications in the development of a great variety of microfluidic systems consisting of valves, pumps, and mixers to be utilized as an efficient method for transporting micro volumes of fluids. Examples of such applications are drug delivery, DNA analysis, and biological/chemical agent detection sensors on microchips [1]. In this context, electroosmosis enables fluid pumping and flow control using electric fields, eliminating the need for mechanical pumps or valves with moving components. Nowadays, these microfluidic devices are being fabricated by using hydrophobic materials, such as polydimethylsiloxane (PDMS) and poly (methyl methacrylate) (PMMA) [2]. While the no-slip boundary condition has been used in order to theoretically analyze electro-osmotic flows, experiments in the past years have demonstrated that the fluid can slip at solid surfaces [3–5]. Huang and Breuer [6] measured the slip length of ionic aqueous solutions by using a total internal reflection velocimetry (TIRV) technique, and they reported slip lengths of approximately 100 nm or less.

Furthermore, molecular dynamic simulations suggest that the conventional non-slip boundary condition becomes invalid because of the hydrophobicity of the solid surface [7,8]. Also, Tandon and Kirby [9] discussed the behavior of water at hydrophobic interfaces from the standpoint of its impact on electrokinetic phenomena in microfluidic devices fabricated with hydrophobic polymers.

The hydrodynamic slip boundary condition was proposed by Navier [10] who assumed that the slip velocity on the solid surface $\boldsymbol{u}_s$ obeys the following relationship:

$$\boldsymbol{u}_s = \lambda_s \boldsymbol{n} \cdot \left[(\nabla \boldsymbol{u}) + (\nabla \boldsymbol{u})^T \right], \tag{1}$$

where $\boldsymbol{u}$ is the fluid velocity profile, λ_s is the slip length, also known as Navier length; $\mathbf{n}$ is the unit vector normal to the surface directed into the fluid. In this direction, as discussed in Bocquet and Barrat [8], Squires [11], and Tandon and Kirby [9], with simple arguments, they present the slipping analog for Smoluchowski's equation, given by

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$$U_{HS}^{s} = -\frac{\varepsilon\zeta E_0}{\mu}\left(1+\frac{\lambda_s}{\lambda_D}\right), \tag{2}$$

where E_0 is the strength of an externally applied electric field, ε is the permittivity, μ is the dynamic viscosity and λ_D is the Debye length. From Eq. (2), it is evident that the Helmholtz–Smoluchowski velocity in a microchannel can be increased according to the ratio λ_s/λ_D; such effect could be of order $10^2 - 10^4$, which would provide extremely efficient microfluidic pumping based on electro-osmotic flow [12]. Evidently, this affects other fundamental transport parameters in electro-osmotic flows such as the volumetric flow rate, temperature and electric fields in a microchannel. In addition, from the same equation (2), since the physical properties can be temperature-dependent, if any of the physical properties changes along the microchannel due to temperature variations, the velocity U_{HS}^{s} would also vary longitudinally. The above argument can be also inferred from Eq. (1), because of the tangential component of the slip velocity on the wall is proportional to the rate of strain at the surface, which could vary as a consequence of temperature gradients along the microchannel. In this manner, Lauga and Stone [13] showed the importance of being very careful when interpreting experimental results with slip boundary conditions, since the viscosity of fluids changes due to viscous heating. Therefore, it is really important to consider the variations of the viscosity with the temperature and the influence it has on the slip boundary condition.

At this point it is necessary to say that the most of investigations regarding electro-osmotic flows with the slip boundary conditions concern to Newtonian fluids. However, in a recent review, Berli [14] discuses relevant consequences of the presence of apparent slip on the dynamics of electrokinetic flow of polymer solutions and its implications. He presents evidences of apparent slip in electrokinetics, showing practical consequences in: i) electrokinetic transport of polymer solutions in microchannels, ii) electro-osmotic pumping and iii) electrokinetic energy conversion. In this context, Afonso et al. [15] investigated the steady-state slip flow of viscoelastic fluids in hydrophobic two-dimensional microchannels under the combined influence of electro-osmotic and pressure gradient forcing with symmetric or asymmetric zeta potentials at the walls. They analyzed the influence of the Navier slip condition on the fluid flow. However, these authors considered uniform properties of the fluid. On the other hand, Park [16] determined the Navier slip coefficient of hydrophobic microchannels by exploiting the streaming potential.

Therefore, the aim of this work it to extend the analysis of electro-osmotic flows of viscoelastic fluids, including the slip boundary condition on the wall of the microchannel, taking into account the presence of the Joule heating effect. To the best author's knowledge, there are no studies regarding electro-osmotic flows by including the slip at the interface fluid-solid, when non-uniform fluid properties in the flow domain are considered. In this work we derive analytical and numerical solutions of electro-osmotic slip flow of Phan-Thien–Tanner fluids, when a uniform electric field is externally imposed, inducing temperature gradients along the microchannel, which modifies the viscosity coefficient, the electrical conductivity and the relaxation time of the fluid. These variations of the properties will affect the slip velocity at the wall and we show that the use of hydrophobic walls tends to diminish the heating of the fluid due to the Joule effect.

2. Formulation

We consider the EOF of a viscoelastic liquid in a slit microchannel of half height H, wall thickness h, and length L, with $L \gg H$, as shown in Fig. 1. The flow is only driven by the effect of electro-osmotic forces caused by an electric field of strength E_0 in the axial direction, which is given by $E_0 = \phi_0/L$, where ϕ_0 is the value of the imposed electric potential at the inlet of the microchannel. The microchannel is supported at the two ends by two isothermal liquid reservoirs, which are found at temperature T_0 and pressure P_0 [17,18]. In addition, a uniform heat loss q_0'' is imposed at the external surface of the micro-channel wall for $0 \leq x \leq L$, and thus, the temperature of the fluid varies in both transversal and axial directions. Both ends of the wall of the microchannel are considered, in a first approximation, adiabatic; this assumption is based on the fact that the heat losses in both ends are really less than the heat loss q_0''. It is assumed that the ζ-potential at the shear surface between the charge surface and the electrolyte solution is axially invariant. Owing to the geometry, a 2D rectangular coordinate system (x,y) is adopted with the origin at the microchannel inlet and x-axis along the microchannel centerline.

In addition, other assumptions on which the mathematical model is obtained are the following: (i) the charge in the electric double layer (EDL) follows a well-known Boltzmann distribution, and the external voltage is significantly larger than the streaming potential induced by the flow; (ii) wall potentials are considered small ($e\zeta/k_BT \ll 1$, typically $\zeta < 25$ mV) so that the Debye–Hückel linearization holds valid; otherwise, the limit of high zeta potential must be taken into account; (iii) The applied electric field is weak, i.e., $\phi_0/L \ll \zeta/\lambda_D$, where ϕ_0 and λ_D are the potential difference of the applied electric field and the Debye length, respectively; this assumption implies that the applied electric field distorts the structure of the EDL by a negligible amount [19]; (iv) the fluid is assumed as a symmetric $(z{:}z)$ electrolyte solution; (v) viscosity function, relaxation time and electrical conductivity of the fluid are considered temperature-dependent, because these are the most sensitive properties to temperature variations [17]; these physical properties are defined as $\eta = \eta_0\exp[-b(T_f - T_0)]$, $\lambda = \lambda_0\exp[-b(T_f - T_0)]$ [20,21], and $\sigma = \sigma_0[1 + a(T_f - T_0)]$ [22], respectively. Here η_0, λ_0 and σ_0 are the viscosity, the relaxation time and the

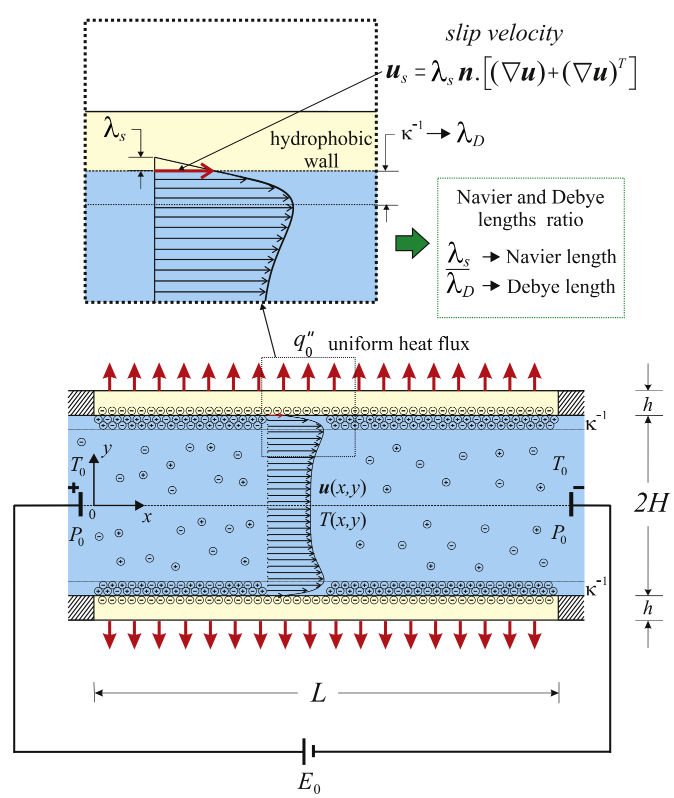

Fig. 1. Schematic diagram of the studied physical model.