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Numerical study of melting a rod by a periodically moving local heat source

Vadim Mizonov [a], [*], Nicolay Yelin [b], [1]

[a] Department of Applied Mathematics, Ivanovo State Power Engineering University, Rabfakovskaya 34, 153003, Ivanovo, Russia
[b] Department of Hydraulics, Heat and Mass Transfers, Ivanovo State Polytechnic University, 8 Marta 20, 153037, Ivanovo, Russia

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ABSTRACT

The objective of the study is developing a simple yet informative mathematical model that describes the kinetics of melting a rod under the action of a localized periodically moving heat source. For this purpose, a cell model is used with the heat conduction matrix that takes into account different properties of liquid and solid phases of the rod material. Zones of the rod that are outside of the local heat source action have heat exchange with the outside environment. It is shown that the melting kinetics strongly depends on the program of heat source motion along the rod and on its residence time at each of its positions. The optimal program of heat source motion that allows melting the whole rod over the shortest possible period of time is found.

1. Introduction

The problem in question arises in some technologies when there is a need to heat up and melt an object by a heat source whose dimensions are much smaller than those of the object to be melted. The simplest example is defrosting a finite fragment of a water pipe full of frozen water by a burner. Sometimes (for instance, at a low outside temperature), this cannot be done if the heat source is only applied to one particular point of the fragment, and it is only its motion along the pipe that can solve the problem. In this case, the complete defrosting time strongly depends on the program of the burned motion over the pipe length – which can be the objective function of investigation. Analogous problems can arise in other technologies, for example, in hand treatment of roof materials by a gas burner, in soldering, and in additive technologies when laser beam treatment of thin layer of fine powder occurs.

This problem is related to the non-linear heat conduction problems with a localized moving heat source. Its non-linearity is conditioned by the phase transformation of melting/solidification type and various thermal effects connected with it. Different aspects of the problem solution can be found in literature. However, most of them are connected with the analytical or semi-analytical approach to solving this problem – which inevitably implies using far-going assumptions.

A solution to the problem of heat conduction in a rectangular plate exposed to a moving heat source was presented in Ref. [1]. The heat source moved along an elliptical trajectory that always remained within the boundaries of the plate area. The exact solution to the problem in an analytical form was obtained by applying the Green's function method. Exemplary results of numerical calculations to determine the temperature distribution in the plate were presented. However, first of all, this was the only trajectory of the source motion and, second, the phase transformation was not taken into account. The problem of the object being heated by a moving source with application to welding was also examined in Ref. [2] where the meshless local Petrov-Galerkin method was developed. The problem of bead-on-plate welding (a moving heat source problem), and the fundamental properties of the method were investigated to verify the applicability of the proposed method. The investigation was mainly oriented to the influence of the number of nodal points on the accuracy of the solution. The issue of the influence of the program of the source motion on the

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[*] Corresponding author. Tel.: +7 910 9948858; fax: +7 4932385701.
E-mail addresses: mizonov46@mail.ru (V. Mizonov), yelinnn@mail.ru (N. Yelin).

[1] Tel.: +7 910 9810020.

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process properties was beyond this particular study. The study presented in Ref. [3] describes the analytical and numerical solution of the heat conduction equation for a localized moving heat source of any type to be used in laser material processing, such as welding, layered manufacturing and laser alloying. The analytical solution for a uniform heat source was derived from the solution of an instantaneous point heat source. The result was evaluated numerically and was compared to existing solutions for the moving point source and a semi-ellipsoidal source. Next, the result was used to demonstrate how such model can be used to study the effect of the heat source geometry. To investigate the effect of the temperature dependence of the material parameters, in particular the latent heat of fusion, a finite difference model was implemented. In spite of the fact that many components of the process were taken into account the influence of the program of the source motion remained beyond the scope of attention of the study. An analytical approach of transient heat conduction in two-layered material, of finite depth, with an imperfect thermal contact, subjected to a moving gaussian laser beam was developed in Ref. [4]. The method consists in deriving the solution of the homogeneous part of the heat equation by using the well known separation of variables method and expressing the source term in series form. This model has been successfully applied on a practical system; laser cladding of electronic copper tracks on alumina substrates. This analytical model can also be used for estimation of the thermal contact resistance between the layers. However, the model does not take into account the phase transformation. An analytical method of computation of temperature field in a half-infinite body caused by heat source with changeable direction of motion was presented in Ref. [5]. Analytical temperature field was approximated by straight segments for volumetric heat source with a trajectory considering temperature changes caused by next transitions (increase in temperature connected to action of the heat source and self-cooling of areas heated-up earlier). In that instance computations were carried out for cuboidal elements made of steel for various heat trajectories. However, the phase transformation in the body was not taken into account too. Later on, in the paper [6], the melting process was introduced in the model but only as a reason of the increase of heat transfer from the heat source to the body. Melting of the body itself was not examined. A model that describes the transient heating of a thin wire causing the tip to melt, roll-up of the molten mass into a ball due to surface tension forces, and the subsequent solidification of the molten material due to conduction up the wire and convection and radiation from the surface, was proposed in Ref. [7]. The wire was assumed to be heated at its lower tip to a temperature beyond the melting temperature of the wire material by heat flux from an electrical discharge. However, the objective of the study was formation of the drops of melt when the heat source was localized at the edge of the wire. Despite the obtained results were approved experimentally in Ref. [8], the approach can be hardy applied to solve the problem in question.

However, even if an analytical solution to a heat conduction problem is formally obtained, several problems arise between it and its engineering application. There should be a computational algorithm that transforms the obtained formulae into numerical results, and the algorithm is far from being correct and precise. In order to solve this problem the concept of intrinsic verification of analytical solutions of heat conduction problems found in books or another databases was describes in Ref. [9].

In Ref. [10], the moving mesh method was used to simulate the blowup in a reaction–diffusion equation with traveling heat source. It was shown that the finite-time blowup occurred if the speed of the movement of the heat source remained sufficiently low, and the blowup procedure was not fixed at one point not like that for stationary heat source. In this simulation, a new moving mesh algorithm was designed to deal with the difficulty caused by the delta function in the traveling heat source. The convergence rates were verified and new blowup figures were generated from the numerical experiments. In Ref. [11], an analysis for simulating melting heat transfer around a moving, horizontal, and cylindrical heat source is presented. Motivated by the experimental observations, the melt domain was divided into two regions, namely, the close-contact region and the melt pool region. Two mathematical models were formulated and solution procedures were developed accordingly. The temperature and the flow fields in the two regions are calculated for a constant surface temperature heat source, and the resulting velocity of the source and motion and shape of the interface are determined. The effects of the prescribed surface temperature of the source and its density, as well as influence of natural convection in the melt pool, were investigated and reported. The predicted melt flow structure and the motion and shape of the solid–liquid interface are found to be in good agreement with the experimental observations when natural convection in the melt is included in the model. Thus, the melting process in a close proximity of the moving heat source was examined but not in the whole object.

The analyses of these and other works show that despite the fact that many solutions for different aspects of the problem were obtained on a high mathematical and physical level, they all are rather far from direct engineering needs. The main tasks of engineering interest are the following. Can a rod be completely melted by a localized stationary heat source? Can this be done by a moving heat source? What program of the heat source motion does allow a complete meltdown of the rod over the shortest period of time?

A cell model of heat conduction that formally uses the mathematical tools of the theory of Markov chains to solve the problems is proposed below. It was successfully used to describe the processes in particle technology [12,13] and heat and mass transfer in technological equipment [14,15]. Its application allows freeing the solution from necessity to make far-going assumptions in order to obtain it that often decreases model adequacy. Its application allows us to get rid of the necessity to make far-going assumptions – which often impairs the model's adequacy.

2. Mathematical formulation of the problem

The object of modeling is shown schematically in Fig. 1. It is a one-dimensional rod of the unit cross section, heat insulated at the edges. Its side periphery is fully or partly open to the heat exchange with outside medium. A localized heat source is applied in the point x_s than can travel over the rod length. A program of its motion $x_s(\tau)$ is given in advance. If the local rod temperature reaches the melting point t_{me} the phase transformation occurs. It can be melting or solidification depending on the temperature growth or decrease. At

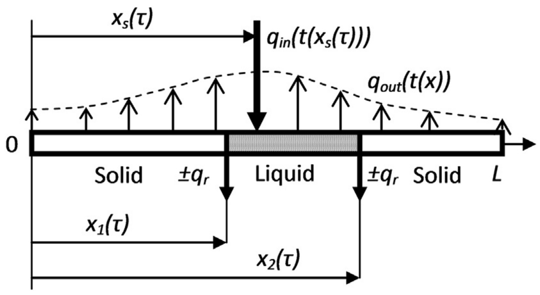

Fig. 1. Schematic presentation of the process.