



## Review

# Non-Fourier effects in macro- and micro-scale non-isothermal flow of liquids and gases. Review



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## ABSTRACT

Non-Fourier heat transfer is reviewed and examined for fluids possessing significant thermal relaxation and retardation times characterizing the response of the heat flux and the temperature gradient to changes in one another. Non-Fourier heat transport occurs in a wide range of applications, including superfluid helium, fluids subjected to rapid heating, and strongly confined fluids. For nanofluids both the relaxation and retardation times are expressed in terms of nanoparticle concentration and suspension properties. The parallels between non-Fourier fluids and viscoelastic polymeric solutions are established. For viscoelastic fluids, the constitutive equation for stress must be frame invariant, a condition that must also hold for the constitutive equation for heat flux. This survey examines a variety of conditions and flow configurations under which non-Fourier effects may be significant in the non-isothermal flow of both gases and liquids.

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## 1. Introduction

Heat transfer is commonly governed by the classical Fourier constitutive law. When used in combination with the First Law of Thermodynamics, Fourier's law predicts an infinite speed of heat transport, in the sense that the effect of a thermal disturbance at a given point is instantaneously observed elsewhere. In reality, however, heat is transferred by molecular interactions [38], and so will travel through a material at a finite speed. This behaviour can be characterized by the Maxwell-Cattaneo or Cattaneo-Vernotte (CV) equation, in which Fourier's law for heat transport is modified by adding a transient term equal to the partial time derivative of the heat flux multiplied by the thermal relaxation time of the medium. The thermal relaxation time is the characteristic time for the heat flux to reach a new steady state following a perturbation to the temperature gradient. In contrast to Fourier's law, the CV equation is hyperbolic, and so predicts wave-like behaviour for the temperature and heat flux. The relaxation time can represent a time delay in the heat flux whenever a temperature gradient is imposed; this is the case of single-phase-lagging (SPL) media. If, in addition, the

temperature gradient experiences a delay (retardation) relative to the heat flux, the medium is designated as a dual-phase lagging (DPL) medium [82].

Non-Fourier effects have long been known to exist in the form of *second-sound* thermal waves in superfluid helium [23,50]. More recently, they have been observed in a variety of phenomena involving ultrafast heating and complex fluids. Most common practical problems involve materials with relaxation times on the order of picoseconds [15]. When this relaxation time is much shorter than the reciprocal of the heating rate, the CV equation reduces to the classical Fourier model. However, there are systems in which the relaxation time may not be considered negligible. Noteworthy examples include some acids and biological tissues, which may possess relaxation times between 1 and 100 s [15]. On the other hand, processes such as laser pulse heating introduce a large quantity of energy over a short time, which can be comparable to the relaxation time of the medium. In some applications, the duration of the laser pulse is on the scale of femtoseconds [48], which is on the order of the electronic relaxation time [10]. When the time scale of the heat transport is of the same order of magnitude as (or faster than) the thermal relaxation time, non-Fourier effects become significant. The importance of non-Fourier effects can be characterized by a dimensionless ratio of time scales. If  $D$  is a typical length scale of the system,  $\kappa$  is the thermal

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diffusivity of the fluid, then the thermal diffusion time is  $D^2/\kappa$ . The Cattaneo number  $C$  is the ratio of the thermal relaxation time  $\tau$  to this thermal diffusion time:  $C = \tau\kappa/D^2$ . Interestingly,  $C$  increases relatively rapidly as  $D$  decreases, so non-Fourier effects are expected to become significant in very narrow systems, such as convection and flow in micro- and nano-devices [6,56,61,74].

It has long been realized that adding the transient term to Fourier's law does not completely address the problem of instantaneous thermal relaxation [35,46,84]. The CV equation is not a frame-invariant constitutive relation and, as such, is restricted to non-deformable media. Consequently, thermal convection problems, for instance, cannot be treated using the CV equation, by simply adding the convective term to the rate of change of the heat flux. Several objective derivatives have been used in the equation in an effort to remedy this situation. However, they each have had their own shortcomings. The most promising modification studied to date is that of Christov [19]; whose use of the Oldroyd upper-convected derivative leads to the frame indifferent CV equation, which will be referred to as the upper-convected CV equation. This model was recently revisited by Khayat & Ostoja-Starzewski [40]. Coupled with the energy equation, this constitutive equation can also yield a single equation for the temperature field, an advantage that other invariant formulations do not possess [19]. This equation replaces Fourier's law for heat flow whenever the relaxation time is relevant (i.e., when  $C$  is not small), and reduces back to Fourier's law when  $C = 0$ .

Although a large number of studies were devoted to non-Fourier effects in conduction, there is limited literature available on non-Fourier heat transport involving flow. For thermal convection, much of existing work pertains only to thermal instability [20,46,58,75,76]. This is rapidly changing, however, with the growing importance of fast processes, micro- and nano-fluidics, and the flow and convection of nanofluids, which are suspensions of nanoparticles in a normal (base) liquid. The addition of nanoparticles to the base fluid is believed to increase the thermal relaxation time [89], and exhibit non-Fourier effects. Rather than treating heat transfer in the two-phase fluid-nanoparticle system explicitly, an alternative approach is to consider a homogeneous fluid with non-Fourier heat transport, resulting from a finite relaxation time. This approach is similar to that used in modelling viscoelasticity in polymeric solutions and colloidal suspensions, in which the addition of solute/particles leads to a finite stress relaxation time [7]. Non-Fourier heat transport in nanofluids has been discussed extensively in the literature [83,88,89,92]. Wang & Wei [92] established the equivalence between the transport of heat in a two-phase system, with Fourier heat transport in each individual phase, and non-Fourier heat transport in a homogeneous DPL fluid. The resulting DPL constitutive equation for heat conduction is of the non-Fourier type, relating the temperature gradient to the heat flux with lagging relaxation and retardation times. Non-Fourier heat transport also arises in other two-phase systems. In particular, heat is transported by thermal waves known as second sound in superfluid liquid helium, which is usually treated using a two-fluid model accounting for normal fluid and superfluid components [23]. The equivalence between the two-phase and DPL formulations allows an alternative approach to the traditional two-phase approach. The importance of non-Fourier effects depends on the value of the relaxation, which depends, in turn, on the properties of the two-phase medium. Possible advantages of the DPL approach include easier implementation of experimentally measurable and/or controllable initial and boundary conditions in heat flux and temperature, reduction in the number of fluid parameters, more manageable analytical solution for the steady base state, and a wider range of potential applications. The DPL approach will be revisited in section 5, to include

convective effects, generalizing the existing SPL model [19,40]. The DPL formulation applies to a variety of phenomena [36,66,84], and not just nanofluids.

## 2. Governing equations for non-Fourier heat transfer

While conservation equations are straightforward to establish and are valid for any fluid, constitutive equations for stress and heat flux are in general difficult to derive for complex fluids, and, more generally, for situations where Newton law of viscosity and Fourier's law of heat do not apply. In this section, we review the general conservation equations, and elaborate on the constitutive equation for heat flux. The treatment is restricted to Newtonian fluids. The parallel between non-Fourier heat transport and non-Newtonian effects is discussed in section 5.3.

### 2.1. Constitutive equation for heat flux

When the thermal relaxation time  $\tau$  is comparable to the thermal process time, Fourier's law for heat transport ceases to be valid. In this case, the thermal relaxation time can be treated as a delay time, with the temperature gradient  $\nabla T$  at a material point  $\mathbf{X}$  and time  $t$  being related to the heat flux vector  $\mathbf{Q}$  at the same point, but at a delayed time  $t + \tau$ . In this case, one can approximately represent the delay by the relation:

$$\mathbf{Q}(\mathbf{X}, t + \tau) = -k\nabla T(\mathbf{X}, t). \quad (2.1)$$

Here  $k$  is the thermal conductivity of the medium. The delayed flux relation (2.1) is also known as the single-phase-lag (SPL) model [82], and should be interpreted with caution. In fact, it is physically incorrect, and should not be regarded as an appropriate constitutive relation, as we shall argue below. In the absence of flow, the constitutive equation that incorporates correctly the time delay or heat inertia is the Cattaneo-Vernotte (CV) equation [91]:

$$\tau \frac{\partial \mathbf{Q}(\mathbf{X}, t)}{\partial t} + \mathbf{Q}(\mathbf{X}, t) = -k\nabla T(\mathbf{X}, t). \quad (2.2)$$

It is tempting to assume that equation (2.2) is obtained by expanding (2.1) in a Taylor series and keeping only the leading-order term in  $\tau$ . From a fundamental perspective, this is misleading. To see this, consider the energy equation in the absence of flow (assuming no heat source):

$$\rho c_v \frac{\partial T(\mathbf{X}, t)}{\partial t} + \nabla \cdot \mathbf{Q}(\mathbf{X}, t) = 0, \quad (2.3)$$

where  $\rho$  is the density and  $c_v$  is the specific heat at constant volume. Upon using relation (2.1), the following equation for the temperature field is obtained from (2.3), namely

$$\frac{\partial T(\mathbf{X}, t + \tau)}{\partial t} = -\kappa \nabla^2 T(\mathbf{X}, t). \quad (2.4)$$

Note that equation (2.4) is the classical heat (diffusion) equation with a time delay, and is not equivalent to system (2.2) and (2.3). In fact, equation (2.4) suffers from important limitations, such as instability with respect to initial conditions [98]. Similar limitations or anomalies are also encountered for the dual-phase-lag (DPL) model, which will be introduced in section 5 below (see, for instance, [60,67]).

In a fluid, heat can also be transported by flow. This can be accommodated in equation (2.2) by reformulating the equation and rendering it objective or frame invariant. This was achieved by Christov [19], and later revisited by Khayat & Ostoja-Starzewski

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