International Journal of Thermal Sciences 95 (2015) 136–146

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Nonlinear pattern selection and heat transfer in thermal convection of a viscoelastic fluid saturating a porous medium

S.C. Hirata, G. Ella Eny, M.N. Ouarzazi*

Laboratoire de Mécanique de Lille, UMR CNRS 8107 – Université Lille I Bld. Paul Langevin, Villeneuve d'Ascq Cedex 59655, France

ARTICLE INFO

Article history:
Received 14 November 2013
Received in revised form
16 October 2014
Accepted 4 January 2015
Available online 7 February 2015

Keywords:
Viscoelastic fluid
Thermal convection
Nonlinear stability
Porous medium

ABSTRACT

The linear and nonlinear dynamics of convection in porous media heated from below and saturated by a viscoelastic fluid are examined. Two geometrical configurations are considered. For infinite layer, it is found that stationary structures (SS), traveling waves (TW) or standing waves (SW) can be stable at the onset of convection, depending on the viscoelastic properties. For weakly viscoelastic fluids, we find that SS are the preferred convective pattern. Otherwise, diluted polymers promote TW while concentrated ones favors SW. The heat transfer associated to the three types of convective patterns are evaluated and compared. For a square box, an amplitude equation is derived in the vicinity of the codimension-two bifurcation point where both the stationary and oscillatory instabilities occur simultaneously. The dynamics associated with the interaction between the two kind of the instability is studied. In particular when oscillatory convection appears first, we found that a secondary nonlinear transition to stationary convection may occur. This prediction is in accordance with the few existing simulation results. Possible connections between experiments and our findings are discussed.

1. Introduction

The classic stability problem of convection in a porous medium heated from below and saturated by a Newtonian fluid has been extensively investigated in the past owing to its major importance in many natural and practical applications (for detailed reviews, see Nield and Bejan [1]). The instability of the conduction state occurs as a result of the buoyancy effect due to heating. Stationary bifurcation takes place when the Rayleigh number, which is a dimensionless measure of the temperature difference across the layer, exceeds a critical value. Recently, there has been a continuously increasing interest to the corresponding problem in the case of viscoelastic fluids. The study of such fluids have applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, suspension solutions and petroleum activities. In rheology, one crucial problem is the formulation of the constitutive equations regarding viscoelastic fluid flows in porous media. For a Darcy-Maxwell model, there is an additional dimensionless parameter besides Ra, namely the relaxation number λ_1, which represents the stress relaxation time produced by an elastic response to strain, scaled by the thermal diffusion. The general model of Oldroyd-B includes a strain retardation time as well as a stress relaxation time, thus introducing a new dimensionless parameter λ_2. The competition between the processes of stress relaxation, strain retardation and thermal diffusion may lead to either an oscillatory or a stationary convective instability as the first bifurcation depending on the λ_1 and λ_2 values.

In most theoretical works [2–11] the porous layer is assumed to be of infinite horizontal extent. In that case, weakly nonlinear theory has been used by Kim et al. [2] and Zhang et al. [6] to study the temporal evolution of standing waves at the onset of oscillatory instability. With infinite horizontal porous cavity, traveling waves are also possible. The question of whether standing or traveling waves are preferred at onset has not been fully addressed. Therefore, one of the objectives of the present work is to provide a full investigation of the stability of standing and traveling waves near the onset of oscillatory convection.

However since experiments can only take place in finite containers, the comparison between theoretical and experimental results is not always an easy task as sidewalls play an important role in small porous boxes. From the linear point of view, stability analysis of the problem of two dimensional convection in a bounded porous cylinder [12] and in rectangular porous containers [13] showed that the spatial horizontal pattern of the convective cells is

* Corresponding author.
E-mail addresses: silvia.hirata@univ-lille1.fr (S.C. Hirata), ella.eny@univ-lille1.fr (G. Ella Eny), najib.ouarzazi@univ-lille1.fr (M.N. Ouarzazi).

http://dx.doi.org/10.1016/j.ijthermalsci.2015.01.002
1290-0729/

no longer degenerate as in infinite layers but is determined by the lateral confinement of the sidewalls. As far as the authors are aware, the numerical investigation of convection in a porous square box saturated with viscoelastics fluids conducted in Ref. [4] is the only published paper focusing on the evolution of flow bifurcations and heat transport at high Darcy–Rayleigh numbers. For some combinations of the relaxation and retardation times, these numerical simulations reveal that in the region of viscoelastic parameters where the oscillatory instability develops first, the system may undergo a secondary nonlinear transition to stationary patterns. Consequently the heat transport estimated by the Nusselt number is drastically modified in the supercritical region where the system exhibits this secondary bifurcation. It is interesting to recall that this kind of nonlinear transition from oscillatory patterns to stationary ones was also observed in experiments conducted by Kolodner [14] who examined the convection of buffer solutions of long DNA suspensions in an annular fluid cavity.

In the present paper, we propose a bifurcation analysis motivated by a desire to shed new light on the numerically [4] or experimentally [14] observed convective patterns transition. We demonstrate that the proximity of oscillatory and stationary instabilities near the codimension two (CT) point may lead to their interaction such that the resulting temporal convective pattern may significantly differ from the patterns that would appear far from the CT point. We proceed by reconsidering linear stability analysis in a square porous cavity in order to locate on the parametric space the CT point where both the steady and Hopf bifurcations occur simultaneously. In the vicinity of this point we reduce the full dynamic equations describing the system to time-dependent amplitude equation which allows one to study the nonlinear interaction between the two kind of the instability.

2. Linear analysis

We consider a isotropic and homogeneous bounded three-dimensional porous layer of height H and horizontal rectangular section with dimensions aH and bH, in which $b \ll 1$. The porous medium is saturated by an Oldroyd-B fluid and we assume that the solid matrix is in local thermal equilibrium with the fluid. The lower and upper horizontal impermeable walls are kept at constant temperatures T_0^* and T_1^* $(<T_0^*)$ respectively, while the four vertical walls are considered impermeable and adiabatic. We use the Darcy law extended to viscoelastic fluids to describe momentum transfer and we assume that the Oberbeck Boussinesq approximation holds. As in Refs. [10], we choose H, $H^2(\rho c)_{sf}/\lambda_{sf}$, $T_0^* - T_1^*$, $\lambda_{sf}/(H(\rho c)_f)$, and $\lambda_{sf}\mu_f/(K(\rho c)_f)$ as reference quantities for length, time, temperature, filtration velocity, and pressure. With this scaling, the following set of dimensionless perturbation (around the conduction state) equations is obtained:

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

$$\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 w - Ra\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\nabla_h^2 \theta = 0 \tag{2}$$

$$\frac{\partial \theta}{\partial t} - w + (\vec{u} \cdot \nabla)\theta - \nabla^2 \theta = 0 \tag{3}$$

where ∇_h^2 is the horizontal laplacian, and the dimensionless parameters are: the filtration Rayleigh number $Ra = (K\alpha_f gH(T_0^* - T_1^*)(\rho c)_f)/(\lambda_{sf}\nu)$, the relaxation time $\lambda_1 = (\lambda_1^* \lambda_{sf})/(H^2(\rho c)_{sf})$ and the retardation time $\lambda_2 = (\lambda_2^* \lambda_{sf})/(H^2(\rho c)_{sf})$. The rheological model for viscoelastic fluids, such as a polymeric solution composed of a Newtonian solvent and a polymeric solute of viscosities μ_s and μ_p respectively yields $\Gamma = \lambda_2/\lambda_1 = \mu_s/(\mu_s + \mu_p)$ [15]. The ratio Γ may also be used as a parameter instead of λ_2.

Here, α_f, λ_{sf}, (ρc), K, and μ_f are respectively the thermal expansion coefficient, the thermal conductivity, the heat capacity per unit volume, the permeability of the porous medium, and the dynamic viscosity of the fluid. Subscript (sf) refers to an effective quantity. The physical boundary conditions of the velocity field read

$$w = 0 \text{ at } z = 0;1, \quad u = 0 \text{ at } x = 0;a, \quad \text{and} \quad v = 0 \text{ at } y = 0;b. \tag{4}$$

and for the fluctuations of temperature we consider the boundary conditions

$$\theta = 0 \text{ at } z = 0;1, \quad \frac{\partial \theta}{\partial x} = 0 \text{ at } x = 0;a, \quad \text{and} \quad \frac{\partial \theta}{\partial y} = 0 \text{ at } y = 0;b. \tag{5}$$

In the case where $b \ll 1$, the stability problem can be simplified into a two-dimensional one [13].

2.1. Extended cavity

If the porous cavity is supposed infinite in horizontal extent $(a \gg 1)$, the two-dimensional infinitesimal perturbations verifying the boundary conditions can be expressed as usual in normal modes

$$\begin{aligned} w &= w_1 \sin(\pi z)\exp(ikx + \sigma t) \\ u &= u_1 \cos(\pi z)\exp(ikx + \sigma t) \\ \theta &= \theta_1 \sin(\pi z)\exp(ikx + \sigma t) \end{aligned} \tag{6}$$

where k is the wave number and $\sigma = \sigma_r + i\omega \in \mathbb{C}$. The real σ_r is the temporal growth rate, while ω represents the frequency of the oscillations. Therefore the neutral temporal stability curve is obtained for $\sigma_r = 0$ which selects dominant modes at the onset of convection.

Substitution of (6) into the linearized version of (1)–(3) leads to the following dispersion equation:

$$D_\phi(k,\omega) = \left(-B_2\omega^2 - B_1 i\omega + B_0\right) = 0, \tag{7}$$

where

$$B_2 = \lambda_2\left(k^2 + \pi^2\right), \tag{8}$$

$$B_1 = \lambda_2\left(k^2 + \pi^2\right)^2 + \left(k^2 + \pi^2\right) - Rak^2\lambda_1, \tag{9}$$

$$B_0 = \left(k^2 + \pi^2\right)^2 - Rak^2. \tag{10}$$

Equation (7) allows for two different types of instability. For $B_0 = 0$, a stationary instability occurs along the curve $Ra^{(s)} = \frac{(k^2+\pi^2)^2}{k^2}$. The critical value is obtained for $k_c^s = \pi$, so that $Ra_c^s = 4\pi^2$. For $B_0 > 0$ and $B_1 = 0$, we obtain a Hopf bifurcation along the curve

$$Ra^o = \frac{(k^2 + \pi^2) + \lambda_2(k^2 + \pi^2)^2}{k^2\lambda_1} \tag{11}$$

The critical wave number obtained by minimizing Ra^o with respect to k is