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Experimental and computational study of scalar modes in a periodic laminar flow

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A B S T R A C T

Scalar fields can evolve complex coherent structures under the action of periodic laminar flows. This comes about from the competition between chaotic advection working to create structure at ever finer length scales and diffusion working to eliminate fine scale structure. Recently analysis of this competition in terms of spectra of eigenfunctions of the advection–diffusion equation (ADE) has proven fruitful because these spectra contain both fundamental information about how mixing processes create emergent Lagrangian coherent structure and also clues about how to optimize flows for heat and mass transfer processes in industry. While theoretical and computational studies of ADE spectra exist for several flows, experiments, to date, have focused either solely on the asymptotic state or on highly idealized flows. Here we show a coupled experimental and computational study of the spectrum for the scalar evolution of a model of an industrially relevant viscous flow. The main results are the methods employed in this study corroborate the eigenmode approach and the outcomes of different methods agree well with each other. Furthermore, this study employs a Lagrangian formalism for thermal analysis of convective heat transfer in the representative geometry to determine the impact of the fluid motion in the thermal homogenization process. The experimental/numerical methods and tools used in the current study are promising for further qualitative parameter studies of the mixing/heat transfer characteristics of many inline mixers and heat exchangers.

1. Introduction

Classification of industrial heat transfer processes with respect to their ultimate goals admits the application of goal-oriented methods for the investigation of thermal phenomena. In literature these processes are analyzed in two groups: rapid thermal homogenization processes and heat transfer processes with high transfer rates in inhomogeneous directions [1]. Examples of thermal homogenization processes are the production of foods, polymers, steel and glass whereas heat treatment of certain polymers is an example for the latter class of processing.

Thermal homogenization is mainly the evolution of the temperature field from its non-uniform initial state towards the final homogeneous state where the evolution to the final state is governed by the balance between advection and diffusion. Advective-diffusive transport of passive scalars in both time-periodic and spatially-periodic flow fields have been studied extensively [2–9] since the pioneering work of Pierrehumbert on 'strange eigenmodes' [10]—periodic modes with highly complex spatial structure in the limit of zero-diffusivity. These studies are mainly focused on the decomposition of an advecting-diffusing scalar field into its spatial and temporal components: spatial patterns are persistent (and repeating in the case of a time-periodic flow field), whereas temporal evolution is the exponential decay of intensities from a non-uniform initial state with high variance toward a fully uniform state (with zero variance). The most-persistent spatial patterns governing the asymptotic scalar transport, so-called 'dominant eigenmodes', which in the limit of arbitrarily small diffusivity coincide with the 'strange eigenmodes', are the slowest decaying eigenmodes of the advection–diffusion operator and can be found by the decomposition of the linear operator into its eigenfunction–eigenvalue pairs without the

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Nomenclature

$\mathcal{A}$	mapping matrix
A	area, m^2
C	scalar field
$\mathbf{c}$	scalar field in discrete partitions
C'	convective scalar field
$\tilde{C}$	conductive scalar field
$\mathcal{D}$	domain
d	diameter, m
E_k	kinetic energy
F	generated/dissipated internal energy
h	depth, m
$\mathscr{H}_k$	Floquet modes of $\mathscr{L}_2$
k	thermal conductivity, W/(mK)
L	aperture length, m
$\mathscr{L}_2$	advection–diffusion operator
m, p	integers satisfying $\Theta/2\pi = m/p$
P	pressure
Pe	Péclet number
Q	heat transfer rate, W
$\mathbf{Q}'$	net convective flux
$\mathbf{q}$	total conductive heat flux
$\mathbf{q}'$	conductive component of $\mathbf{Q}'$
$\mathbf{q}_c$	convective component of $\mathbf{Q}'$
R	radius, m
Re	Reynolds number
Sr	Strouhal number
St	Stokes number
$\mathcal{T}$	period of motion, $p\tau$
$\tilde{\mathcal{T}}$	dimensionless period
T^*	reference time scale, s
T_1	analytical solution of 2D axisymmetric heat equation, K
T_2	analytical solution of 1D heat equation, K
T_a	typical flow (advection) time scale, s
T_f	fluid particle-wall contact period
T_p	typical response time of particles, s
T_s	forcing period per motion step, s
T_ν	viscous time scale, s
t, t^*	time
U	characteristic velocity, m/s
$\mathbf{u}$	velocity vector
$\mathbf{u}^*$	velocity field with temperature–dependent material properties
$\mathbf{x}$	position vector
α	material (or thermal) diffusivity, m^2/s
Γ	boundary of $\mathcal{D}$
γ_k	expansion coefficients of Floquet modes
Δ	aperture arc angle, rad
$\delta_{\|\mathbf{u}^*\|}$	deviation parameter for velocity
ε	difference in velocity magnitudes of $\mathbf{u}$ and $\mathbf{u}^*$
Θ	offset angle, rad
λ_k	eigenvalues of Floquet operator
μ_k	Floquet exponents, eigenvalues of $\mathscr{L}_2$
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3
τ	period of switching
φ_k	eigenfunctions of $\mathscr{L}_2$

Subscripts

f	fluid
H	horizontal
mw	moving wall
p	particle
sw	stationary wall
s	surface
w	wall
V	vertical
t	top
b	bottom
0	dominant eigenmode

Abbreviations

ADE	advection–diffusion equation
DMD	dynamic mode decomposition
FEM	finite element method
IRT	infrared thermography
PIV	particle image velocimetry
RAM	Rotated Arc Mixer
2D	two-dimensional
3D	three-dimensional

necessity of solving the full advection–diffusion equation (ADE). In the case of an experimental approach, however, the information at hand is the data sequence rather than a mathematical model. Thus, a data processing method capable of capturing the dynamics is necessary to determine dominant eigenmodes and corresponding decay rates of the experimentally acquired time-resolved scalar fields. The dynamic mode decomposition (DMD) is a technique that extracts dynamic information by decomposing the data set into temporal and spatial components such that each mode corresponds to a complex-valued eigenvalue [11,12]. In the present work, the DMD method is employed to determine the eigenmodes and decay rates of both experimentally and numerically acquired scalar fields derived from time-periodic advection.

In contrast, heat transfer processes which necessitate high transfer rates in preferred directions require a Lagrangian approach rather than the use of traditional heat transfer analysis methods based on integrated quantities or empirical correlations. Such an approach enables in-depth analysis of the thermal topology of the heat transfer process and, in turn, optimization of thermal transport routes. The Lagrangian formalism introduced in the work by Speetjens [13] provides a generalized Lagrangian framework for the analysis of heat transfer in which the impact of the fluid motion on the scalar distribution is determined. In this study, the formalism is demonstrated for the temperature field in a representative industrial mixer/heat exchanger, however, it can be applied to any advective-diffusive scalar field. The formalism is also combined with the eigenmode analysis to show that two groups of processes (rapid thermal homogenization processes and heat transfer processes with high transfer rates in inhomogeneous directions) can be analyzed by the same spectral methods.

The current study adopts the Rotated Arc Mixer (RAM) [14] as the representative configuration for in-depth analysis of advective-diffusive transport of scalars in realistic inline mixers. The RAM (Fig. 1) consists of two concentric cylinders; a stationary inner cylinder with consecutive windows that are offset in angular direction and an outer rotating cylinder that induces a transverse flow via viscous drag at the windows as the flow progresses axially with a constant flow rate. A tubular segment bearing a window is