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Non-Boussinesq convection in a square cavity with surface thermal radiation

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ABSTRACT

The interaction of natural convection with surface radiation in a differentially heated square cavity filled with air is considered under large temperature differences. The study has been investigated by direct numerical simulations with a two-dimensional finite volume numerical code solving the time-dependent Navier–Stokes equations under the Low Mach Number (LMN) approximation. Calculations were performed for cases with strong non-Boussinesq effects. The results reveal that the fluid flow and heat transfer are influenced significantly by the surface radiation. At steady state, the top wall is cooled and the bottom wall is heated compared to the case without radiation. The air flow is reinforced near the horizontal walls and the thermal stratification at the core is reduced. The surface radiation reduces the convection heat transfer at the hot wall and increases it on the cold wall.

Transition from steady to unsteady flow has also been investigated. By comparing the solutions in pure convection, the results in combined convection–radiation show that the radiation promotes the occurrence of instabilities leading to an early transition to the unsteadiness and contributes to the modification of the physical mechanism responsible for their onset.

1. Introduction

Problems of natural convection coupled to thermal radiation are encountered in many industrial applications such as furnaces, combustion equipments, burners, cooling of electronic components, printed circuit boards…. Most of these applications proceed with large temperature differences and require a modeling which accounts for realistic fluid properties variations and compressibility effects. Derived from the compressible Navier–Stokes equations, the Low Mach Number (LMN) approximation constitutes an important numerical problem for low speed compressible flows and has the advantage that it presents the same mathematical structure as the incompressible Navier–Stokes equations. In this approximation, the total pressure is split into two terms: a mean thermodynamic pressure which is spatially uniform and depends only on time and a dynamic pressure. Since for Low Mach flows the thermodynamic pressure is very high compared to the dynamic pressure, this decomposition leads to eliminate the acoustic waves which present a severe limitation on the time steps used for numerical integration while large variations of density with temperature are allowed. The interaction between radiation and convection in solid-fluid domains exists in a case of participating media but also by the influence of boundary conditions. The differentially heated cavity problem is a classical case commonly used in the process of CFD codes verification. The aim of this paper is to study the effect of surface thermal radiation on natural convection in a differential heated cavity under strong temperature differences.

Research in radiative–convective heat transfer was investigated in a variety of geometrical configurations and recently they are considered mainly by numerical methods. Several studies concern convection flows in transparent media bounded partially or totally by solid walls: 2*D* or 3*D* rectangular cavities [1–5], closed cavities with a circular or square obstruction inside [6–11], partitioned cavities with baffles [12–14], partially open cavities [15–17], vertical channels [18]. The coupling natural convection, conduction and surface thermal radiation has also been investigated in two [19,20] or three-dimensional rectangular cavities [21].

Among the numerical studies cited above, the physical problem is described by the Navier–Stokes equations under the

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Nomenclature

A	aspect ratio, $A = H/L$
c_p, c_v	specific heat capacities
H	height
L	width
L_0	reference length
N_r	radiation–conduction number, $N_r = \sigma T_0^3 L_0/2\delta\kappa_0$
Nu_{cv}	average convective Nusselt number
Nu_{rad}	average radiative Nusselt number
Nu_g	global Nusselt number, $Nu_g = Nu_{cv} + Nu_{rad}$
$\overline{P}$	mean thermodynamic pressure
P_0	reference pressure, $P_0 = \rho_0 V_0^2$
Pr	Prandtl number, $Pr = \mu_0/\rho_0\alpha_0$
Q_r	dimensionless net radiative flux
Ra	Rayleigh number, $Ra = 2\rho_0\delta g L_0^3/\mu_0\alpha_0$
t_0	reference time, $t_0 = L_0/V_0$
T	non-dimensional temperature
T_0	reference temperature, $T_0 = (T_h + T_c)/2$
V_0	reference velocity, $V_0 = \mu_0 Ra/\rho_0 L_0$
v_x	horizontal component of the velocity
v_z	vertical component of the velocity

Greek symbols

α	thermal diffusivity, $\alpha = \kappa/\rho c_p$
δ	normalized temperature difference, $\delta = \Delta T/2T_0$
κ	thermal conductivity
μ	dynamic viscosity
π	reduced pressure
ρ	density
σ	Stefan–Boltzmann constant
ΔT	Temperature difference, $T_h - T_c$
ε	wall emissivity

Subscripts

0	values at reference temperature
h	hot
b	bottom
c	cold
l	left
r	right
t	top

Boussinesq approximation and a radiation algorithm is used for gray and diffuse surfaces. Note that the Boussinesq approximation is valid only for low temperature differences and considers all the physical properties constant except in the buoyancy force. It results from these studies that radiation influences the heat transfer substantially. For instance, Akiyama and Chong [1] studied the interaction of natural convection with radiation in a square cavity filled with air. Their results have shown that the surface radiation significantly affects the temperature distribution and the flow patterns, especially at higher Rayleigh number. The radiation heat transfer plays an important part in overall heat flux and increases with the increase of emissivity. The convection Nusselt number increases with Ra but little variations were observed with the emissivity. Colomer et al. [2] analyzed the coupling radiation and natural convection in a three-dimensional differentially heated cavity. Both transparent and participating media were explored. For transparent media, the effects of surface radiation on the heat transfer were investigated for different values of the Rayleigh number and a given Planck number. A comparison with a two-dimensional case is presented showing a good agreement between both solutions. Wang et al. [3] studied numerically the interaction of convection with radiation in a two-dimensional differentially heated square cavity. It has been found that the surface radiation reduces the stratification in the cavity core and increases the average Nusselt number with increasing the emissivity. The transition from steady to unsteady flow is also investigated. The results have shown an early transition to the unsteadiness compared to the case without radiation.

Merzhab et al. [6] considered a differentially heated cavity with an inner conducting square body. They found that the radiation exchange homogenizes the temperature inside the cavity and produces an increase in the average Nusselt number as the emissivity increases. Sun et al. [7] considered a square air-filled cavity cooled from below and above, with a heated square body located at the cavity center. The flow structure is investigated for various Rayleigh numbers, emissivity of the walls and sizes of the inner body. Recently, a numerical study has been conducted by Saravanan and Sivaraj [12] in a differentially heated cavity with a heated plate placed horizontally or vertically at its center. It has been found that the convective heat transfer increases with emissivity if the plate is horizontal and decreases if the plate is vertical. The overall heat transfer by convection and radiation increases with the emissivity and the presence of radiation leads to a better homogenization of the temperature within the cavity for both positions of the central plate.

Convection–radiation coupling in the presence of large temperature variations was the purpose of few studies and concerned participating media. The numerical studies performed by Ioan Telega et al. [22], Dubroca et al. [23] and Scarella et al. [24] on various configurations are based on the LMN model. From the literature, it appears that no work was performed on coupled convection with surface radiation under large temperature differences. In this paper two-dimensional simulations under the LMN approximation are reported for a square cavity with gray and diffuse surfaces. Following the previous study of Wang et al. [3], the purpose of this investigation is to analyze the influence of surface radiation on the flow pattern and heat transfer and on the transition from steady to unsteady flows for strong non-Boussinesq regimes. The code validation is performed by comparison with the well-documented results in literature.

2. Problem formulation

2.1. Governing equations

The problem considered here is natural convection in a square cavity placed into a gravitational field $\vec{g}$ which is parallel to the active walls. The inner surfaces are assumed to be diffuse, gray and opaque. The left and right side walls are isothermal at respective temperatures T_h and T_c ($T_c < T_h$), and the bottom and top walls are adiabatic. The cavity is filled with a transparent gas initially at a uniform temperature $T_0 = (T_h + T_c)/2$ and pressure P_0. It is assumed to be an ideal gas with constant specific heat capacities c_p and c_v of ratio $\gamma = 1.4$. Its dynamic viscosity μ and thermal conductivity κ are allowed to depend on temperature. As we are interested in flows induced by large temperature differences, the problem is governed by the Low Mach approximation equations to describe such a flow (Paolucci [25]). The equations are made dimensionless by reference quantities: $L_0 = L$, $t_0 = L_0/V_0$, $T_0 = (T_h + T_c)/2$, $P_0 = \rho_0 V_0^2$, $V_0 = \mu_0/\rho_0 L_0 Ra$. The physical properties (density,