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On exact solutions for anisotropic heat conduction in composite conical shells

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ABSTRACT

In this paper, exact analytical solutions for anisotropic conductive heat transfer in composite conical shells are presented. To the knowledge of authors, the present work is the first exact study in the field of heat conduction in anisotropic cones. The shell has a full conical shape and the fibers are winded around the body. In order to obtain the most general solution, the general boundary condition is considered at the basis of shell and the effect of heat convection resulted from flow motion around the body and different kinds of non-axisymmetric radiative heat flux at outer side of shell is modeled. The exact solution of temperature distribution is obtained via separation of variables method for a general form and some special cases. Due to the existence of a dual second order derivative of temperature in heat transfer equation, it is not possible to obtain directly the temperature distribution using the analytical techniques. In order to cancel this term, the heat transfer equation is transformed to a canonical form using an innovative transformation. The present analytical solution is validated via numerical solution and some applied cases are considered to investigate the capability of current solution for solving the industrial problems.

1. Introduction

Today, applications of composite materials are significantly extended in different branches of industry. These progressive applications are due to the unique properties of composite materials. Composite materials have high ratio of strength to density thus these materials are really appropriate for applications that strength and lightness are major factors. Their significant anti-corrosion features lead to use them in any special places and provides long life for composite structures. Briefly some important applications are considered such as, aerospace elements, heat exchangers, pipelines, sporting goods, brake and friction systems, beams, combustion chambers, pin fins, vessels and biomaterials.

Particularly, composite conical shells are used extensively in aerospace industry as a nose cone but they have some applications in other fields such as naval and civil structures, robots and so on. Most of previous works investigated the mechanical behaviors of composite laminates. In contrast, few works have considered the heat transfer analysis of these materials. The available works about heat transfer analysis usually performed using numerical methods and the contribution of analytical solutions is small. Ma et al. [1,2] used a linear coordinate transformation to simplify the anisotropic problem to an equivalent isotropic one and finally obtained an analytical solution for conductive heat transfer in an anisotropic media. A mathematical formulation for steady-state heat conduction in multilayer bodies has been developed by Haji-Sheikh et al. [3]. They showed that the eigenvalues for homogeneous layers are real but for orthotropic layers could be complex. Onyejekwe [4] using boundary integral theory presented an exact analytical solution for conductive heat transfer in composite media. Blanc and Touratier [5] presented a simple analytical model for conductive heat transfer problems in multilayered structures based on an equivalent single layer approach. Third order polynomial or trigonometric functions in thickness of the laminate and boundary conditions at the top and bottom of the laminate are used and finally three-dimensional solution has been obtained. An analytical solution is obtained by Miller and Weaver [6] that predicts temperature distribution through multiple layers subject to convection and radiation boundary conditions. Huang and Chang [7] presented analytical solution of heat conduction in multilayer composite materials for periodic, unsteady and steady states with use of Green's functions. This method is appropriate for composites of any

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number of layers. Separation of variables method has been used by Singh et al. [8] to find an analytical two-dimensional heat conduction solution in polar coordinate for multilayer medium. An analytical solution of heat conduction in an orthotropic cylindrical fin has been obtained by Bahadur and Bar-Cohen [9]. The results also have been compared with the finite element solution. Lu et al. [10] presented an analytical solution of transient temperature distribution in multi-dimensional composite circular cylinder. In this solution, the boundary conditions are supposed as time-dependent temperature change and also the separation of variables method plays a significant role. Kayhani et al. [11] found an exact analytical solution for steady-state conductive heat transfer of cylindrical composite laminates in $r-\varphi$ directions. This solution is just appropriate for cylinder with high ratio of longitudinal to radial dimension. An analytical solution for heat conduction of cylindrical composite laminate is also presented by Kayhani et al. [12]. The solution is in longitudinal and radial directions and introduces temperature distribution for steady-state condition. Norouzi et al. [13] obtained an exact solution of unsteady conductive heat transfer in cylindrical composite laminates. The Laplace transformation is used to transform the orthotropic heat conduction equation to the frequency domain and finally the separation of variables method is applied to solve the obtained differential equations. Jain et al. [14] presented an analytical solution of multilayer heat conduction in $r-\theta$ spherical coordinate for transient boundary-values. Materials in each layer are isotropic and the solution is appropriate for different kind of homogeneous boundary conditions. An exact analytical solution for steady-state heat conduction in spherical composite laminates is presented by Norouzi et al. [15]. Heat conduction is investigated in $r-\theta$ directions and general linear boundary conditions are used in both inside and outside of laminate. The separation of variables method is used and the final set of equations is solved using the recursive Thomas algorithm. Almost all noted articles try to solve the heat conduction problem in geometries such as: cube, cylinder and sphere but it is really considerable that there are also other important geometries that are significantly applicable in different industries. One of them is conical geometry. Because of complicated geometry comparing mentioned geometries that a cone has, finding an appropriate solution for heat conduction problem is more difficult. Mahishi et al. [16] presented transient heat conduction analysis of a laminated composite nose cone subjected to aerodynamic heating using finite element method. Rubin [17] investigated heat conduction in plates and shells with emphasis on a conical shell. The author used the thermal equations of the theory of a Cosserat surface to find the average temperature and temperature gradient. Materials have been supposed isotropic and the results have been compared with exact solutions. Heat transfer by conduction through a truncated conical shell has been studied by Ray et al. [18]. They used two methods that the first method is based on semi-analytical solution and the second is numerical investigation. From the results for quick and accurate calculation of heat transfer, they found the inner radius of an equivalent cylindrical shell of same thickness.

According to the literature, the previous studies about the conductive heat transfer in composite conical shells are limited to numerical or approximate semi-analytical solutions. The dearth in the literature on exact analytical solution of this problem is related to the complex form of governing equation and the geometry of problem (especially difficulties in finding the suitable coordinate system).

In the present paper, an exact analytical solution of steady-state heat conduction in a composite conical shell is presented for the first time. The cone is supposed as a full cone and the fibers are winded around it with any arbitrary angle. The geometry of conical shell is presented in Fig. 1. According to the Figure, two independent directions x and φ are showed as the components of the coordinate system and also other necessary parameters used in solution process have been indicated obviously. In order to cover wide range of thermal conditions, a general linear boundary condition is applied at the cone base which guarantees the present solution usability in any industrial application.

Here, the heat transfer equation for an element of conical shell is derived using the appropriate coordinate system. The exact solution of heat transfer equation is obtained via separation of variables method. Generally, due to the existence of a dual second order derivative of temperature in heat transfer equation $(\partial^2 T/\partial x \partial \varphi)$, it is not possible to obtain directly the temperature distribution using the separation of variable method. In order to solve this problem, the heat transfer equation is transformed to a canonical form (without any dual derivatives) using an innovative transformation. The presented transformation guarantees applying the harmonic boundary condition in angular direction (φ) and it is shown that finding the temperature distribution and eigenvalues of heat transfer equation is easier by applying it on heat transfer equation. The temperature distributions are also presented for three special cases in which it does not need to use any transformation.

2. Heat conduction in composite materials

In this section, the basic concepts of heat conduction in composite materials are discussed briefly. The general form of Fourier law for conductive heat transfer in orthotropic materials is as follows [19]:

$$\begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} = -\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial T}{\partial x} \\ \dfrac{\partial T}{\partial y} \\ \dfrac{\partial T}{\partial z} \end{Bmatrix} \tag{1}$$

where q is heat flux, k_{ij} is heat conduction coefficient and T is temperature. Generally, on-axis (x_1, x_2, x_3) and off-axis (x, y, z) coordinate systems are defined to solve heat conduction problems in composite materials [20]. Fiber orientation determines the direction of on-axis coordinate system in a way that x_1 is parallel to the fibers direction, x_2 is perpendicular to the fibers direction in layer and x_3 is perpendicular to the layer plane. To study the physical properties in specific directions, an off-axis coordinate system must be specified. Therefore, there is an angular deviation of α between the on-axis and off-axis coordinate systems and also these coordinates are coincident.

Fourier relation for a composite material in on-axis coordinate system is as follows [21]:

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}_{on} = -\begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{22} \end{bmatrix}_{on} \begin{Bmatrix} \frac{\partial T}{\partial x_1} \\ \frac{\partial T}{\partial x_2} \\ \frac{\partial T}{\partial x_3} \end{Bmatrix}_{on} \tag{2}$$

As shown in Fig. 1, α is the angle between the tangent line on cone in x direction and the tangent line in fiber's direction. Applying the rotation by angle α to the on-axis heat conduction coefficient tensor $[k]$, the off-axis conductivity tensor $[\overline{k}]$, is obtained as follows [12]: