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Fully developed laminar mixed convection in uniformly heated horizontal annular ducts

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A B S T R A C T

A Galerkin spectral method has been applied to solve numerically the problem of fully developed laminar mixed convection in horizontal concentric annuli with axially uniform heat flow per unit length, uniform temperature along the periphery of the heated wall, thermal insulation on the remaining surface. Results have been obtained for *PeRa* values up to 10^7, for $Pr = 0.7$, 5 and ∞, for inner to outer diameter ratio $\eta = 0.2$ in case of inner wall heated and for $\eta = 0.2$, 0.4 and 0.6 in case of outer wall heated. Computations allow understanding the onset of secondary flows and their influence on both fluid dynamics and heat transfer. In particular, the distortion of the axial shear stress effects the reduction of the mass flow rate at a given axial pressure gradient. On the other hand, boundary layer thinning determines an increase in the Nusselt number up to a factor 2.5 on average in the investigated range as summarized by suitable correlations. Eventually, results have been compared with other numerical computations and experimental data only available for restricted ranges of the governing parameters.

1. Introduction

Laminar internal flows are often encountered in industrial situations, such as compact heat exchangers, exchangers designed for application in chemical and food processes, cooling of electronic equipment, etc., in which small dimensions and/or low velocities are employed. These flows with heat transfer may be strongly influenced by buoyancy forces arising from temperature differences in the fluid under the influence of the gravitational force field. In fact, experimental and theoretical work evidences that both the effects of natural convection and forced convection may be of comparable order in many cases of practical interest. In this circumstance, the flow orientation with respect to gravity becomes an important parameter. The laminar mixed convection of a fluid in horizontal ducts has been the subject of a number of studies both experimental and numerical, which cover a range of geometrical configurations and boundary conditions. Among the various geometries, the fluid flow and heat transfer between two horizontal concentric cylinders has attracted considerable attention because of its technical importance, as it is present in numerous engineering applications such as gas-cooled electrical cables, and double-pipe heat exchangers, just to mention a few examples. From the analysis of the available literature, it appears that most of the studies are relevant to Newtonian fluids, constant thermo-physical properties, stationary bounding surfaces, and uniform thermal boundary conditions along the girth of the cylinders [1–11]. Only a limited number of authors have considered different conditions such as constant heat flux along the periphery of the inner cylinder [12], forced convection due to a cooled rotating outer cylinder [13], temperature-dependent viscosity [14], thermodependent non-Newtonian fluids [15], and non-uniform circumferential heating [16]. Finally, Chenier et al. [17] performed numerically a linear stability analysis of a fully developed mixed convection flow of air in an annular horizontal duct.

The present study deals with a numerical solution of fully developed laminar flow and heat transfer in horizontal concentric annuli, based on a Galerkin spectral method. The fully developed situation is an asymptotic condition that prevails for large values of the axial coordinates. For purely forced convection in concentric annular ducts both hydrodynamic entrance length and thermal entrance length, the latter for the four fundamental thermal boundary conditions, have been studied by several authors; the obtained results have been compiled by Shah and London and presented in Ref. [18] in tabular form. In the case of combined forced and free convection, only few studies are available on entrance region effect [7–12]. None of them systematically treats

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the determination of hydrodynamic and thermal entrance lengths; only some indication can be found in the case of inner wall heated. Ciampi et al. [7] used water as a working fluid flowing in annuli with η = 0.46 and 0.63. These authors claim that in general a length of 4–6 hydraulic diameters is needed to obtain a linear temperature distribution that is to reach the fully developed region. Islam et al. [10,12], noticed from their experimental runs that with water at 2.14 < *Pr* < 5.34 and 1/η = 1.75, 2.14 the entrance lengths are very small and less than 3.5 times the hydraulic diameter. From the analysis of their numerical results, they conclude that higher *Pr* leads to early development of the flow. Furthermore, the thermal entrance length is relatively smaller for smaller η. However, according to them, it is difficult to identify an entrance length in these cases because the Nusselt number goes through a minimum, then a maximum a finally approaches a constant value. Therefore, while the assumption of fully developed fields is, in general, rather restrictive, only for very short ducts the use of the results presented here is inappropriate.

Among the possible combinations of thermal boundary conditions, the selected ones consist in axially uniform heat flow per unit length, uniform temperature along the periphery of the heated wall, thermal insulation on the remaining surface. Either heating at the inner wall or heating at the outer wall have been considered. The choice of these case studies is motivated both by the lack of complete investigations and by the evidence of significant discrepancies between the results of numerical simulations and experimental data in the available literature. In particular, the numerical study of Nickele and Patankar [5] and the experimental investigation of Ciampi et al. [7] consider only the case of heating at the inner wall. On the other hand, at the best knowledge of the authors, only the paper of Hattori [1], reporting both experimental data, numerical simulations and the perturbation analysis takes into account the case of outer wall heated. Since for heating at the inner wall very good agreement with the findings of Nickele and Patankar has been found, the case of outer wall heated has been studied more deeply, considering a larger number of diameter ratios. Correlations between the relevant dimensionless parameters have been developed as well and compared with the available data and correlations.

2. Problem formulation and governing equations

We consider the steady laminar flow of a Newtonian fluid in a long horizontal annular duct subject to axially uniform heat flow per unit length. Among the various possible thermal boundary conditions, reference is made to uniform temperature along the girth of the inner tube and to adiabatic outer surface and to uniform temperature along the girth of the outer tube and to adiabatic inner surface. According to [18] for this fundamental boundary condition, the wall thermal conductivity is implicitly assumed to be zero in the axial direction and infinite in the peripheral direction. If the local heat transfer coefficient varies along the girth of the heated pipe, as it is the case of this study or for noncircular ducts, it may be difficult to achieve in practice the stated boundary condition. However, it may be considered a limiting case of more realistic boundary conditions and together with the other limiting case of constant heat flux, it is the most frequently investigated fundamental boundary condition in the literature. Referring to the usual Oberbeck–Boussinesq approximation, the thermophysical properties of the fluid are constant except density, which is supposed to be dependent only on temperature according to a linear relationship, in the buoyancy term. Furthermore, the effects of viscous dissipation in the energy equation are neglected.

If we disregard the density variation along the axis, sufficiently far from the duct inlet the problem admits a similarity solution [19] with velocity, pressure and temperature differences in the cross section independent from the axial coordinate. This assumption has been thoroughly discussed by Nieckele and Patankar [5] and it is shown that disregarding the density variation along the axis is justified for values of the ratio between the Richardson number and the Prandtl number less than unity. This condition is easily verified for large Prandtl numbers. On the other hand, for moderate Prandtl numbers, the assumption is justified for small values of the Richardson number. As a consequence, in any case, the Richardson number disappears in the formulation of the problem. In the fully developed region, temperature, T, and pressure, p, through the duct vary linearly with the distance along the axis. That is, in the cylindrical coordinates (r, ψ, z),

$$T(r,\psi,z) = \tau z + T'(r,\psi) \tag{1}$$

$$p(r,\psi,z) = \gamma z + p'(r,\psi) \tag{2}$$

where τ and γ are the axial temperature and pressure gradients, respectively.

Under the stated assumptions and with reference to Fig. 1, the governing equations are then

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \psi} = 0 \tag{3}$$

$$\rho\left(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \psi}-\frac{v^2}{r}\right) = -\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \psi^2}-\frac{2}{r^2}\frac{\partial v}{\partial \psi}\right] + \rho\beta g(T_w - T)\cos\psi \tag{4}$$

$$\rho\left(u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \psi}+\frac{uv}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \psi}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right)+\frac{1}{r^2}\frac{\partial^2 v}{\partial \psi^2}+\frac{2}{r^2}\frac{\partial u}{\partial \psi}\right] - \rho\beta g(T_w - T)\sin\psi \tag{5}$$

$$\rho\left(u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \psi}\right) = -\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 w}{\partial \psi^2}\right] \tag{6}$$

$$\rho c\left(u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial \psi}+w\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \psi^2}\right] \tag{7}$$

where u, v, w denote the velocity components, ρ is the density in the reference state, μ is the viscosity, β the coefficient of thermal

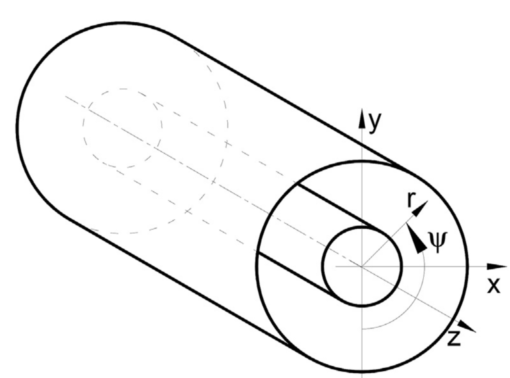

Fig. 1. Geometry and coordinate system.