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Analytical and numerical solution to the convection problem in a shallow cavity filled with two immiscible superposed fluids

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ABSTRACT

Natural convection, in a shallow horizontal cavity filled with two layers immiscible fluids, is studied both numeric and analytic. The horizontal walls of the enclosure are heated by the bottom and cooled by the upper by a uniform heat fluxes while the verticals one are subject to heat fluxes of variable intensity. The analytical solution, based on the parallel flow approximation, is found to be in good agreement with a numerical solution. The existence of both natural and antinatural flows is demonstrated. Indeed, the critical Rayleigh number for the onset of motion is dependent on the side heating intensity. In the case where the upper fluid is more viscous than the bottom one, multiple solutions are possible as well as the a steady states in a range of ratio of the viscosity of the fluids. The fact that one of the two fluids is more heat conductive, affect not only heat transfer generated but also the intensity of flows and the existence of the third cell where we found several solutions. The present model yields explicitly the flow patterns and Nusselt numbers in terms of the governing parameters of the problem.
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1. Introduction

Due to its applications, the natural convection has received large consideration over the past several years in engineering design of advanced technology. These applications span such diverse fields as solar energy collectors, storage tanks, crystal growth, and many other geological, chemical, and astrophysical systems. Generally the preceding work has addressed natural convection in a cavity filled with only one fluid. Comparatively few work which was concluded for the situations where the cavity is filled of two superposed immiscible fluids. However, the study of the above mentioned problem has a direct link with practical situations such as the suggested occurrence of two-layer convection in the Earth's mantle. Physicists have also been attracted by layered convection because of the expected wealth of dynamical behaviours.

The transient development of laminar natural convection in a rectangular cavity containing either one liquid or two immiscible liquids has been studied both numerically and experimentally by Szekely and Todd [1]. Results were obtained for a variety of conditions, including high, low, and intermediate values of Prandtl number. The mechanism of natural convection of two stratified fluid layers was studied numerically by Kimura and al. [2]. The effects of Marangoni convection, resulting from surface tension gradients at the interface, were also considered. It was found that the boundary surface tension promotes or suppresses the convective flow in accordance with the direction of the convective flow. Numerical solutions have been obtained by Myrum and al. [3] for natural convection in the enclosed space between two concentrically cylinders. The enclosed space was occupied by a system of two fluids. It was observed that the flow pattern was dominated by global recirculation zones in each of the fluids, both rotating in the same sense. Natural convection experiments were performed for an enclosure of square cross section containing two immiscible fluids in a layered configuration [4]. Villers and Platten [5], using laser Doppler anemometry, have measured the horizontal velocity profiles in each layer of a system consisting of a combination of water and heptanol. The resulting revealed the existence of three convection cells; not only was there one buoyancy cell in each layer, but there was also a third intermediate convective cell in the water layer. In a subsequent paper the theoretical aspect of the above problem was also considered by these authors [6]. The model, valid

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Nomenclature

A	aspect ratio of the enclosure, L′/H′
a	integer number
CT	dimensionless temperature gradient in x-direction
g	gravitational acceleration, m/s^2
H'	hauteur of enclosure, m
h'_i	thickness of a fluid layer i, m
L'	length of the enclosure, m
Nu	Nusselt number, Eq. (6)
Pr	Prandtl number, υ/α
q'	constant heat flux per unit area, W/m^2
Ra	thermal Rayleigh number
Ra_C^{sub}	subcritical thermal Rayleigh number
t	dimensionless time, $t'/(\sigma H'^2/\alpha)$
T	dimensionless temperature, $(T'-T'_0)/\Delta T'$
$\Delta T'$	characteristic temperature, $q'H'/k$
(x,y)	dimensionless coordinate system, x'/H', y'/H'
(u,v)	dimensionless velocity components, $u'/(\alpha/H')$, $v'/(\alpha/H')$

Greek symbols

α	fluid thermal diffusivity, m^2/s
β	thermal expansion coefficient, K^{-1}
η	dimensionless position of the interface h'_1/H'
λ	thermal conductivity, W/(m K)
μ	fluid dynamic viscosity, Kg/(m s)
υ	fluid kinematic viscosity, m^2/s
Ψ	dimensionless stream function
ρ	density, kg/m^3

Subscript and superscript

C	refers to the center of the fluid
$'$	dimensionless variable
r	relative quantities (layer 2 to layer 1)
i	in the fluid layer i ($i = 1, 2$)
0	reference state

for thin fluid layers, predicts the position and the relative size of the third cell as a function of the various parameters of the problem. Similar results have been obtained by Wang and al. [7] for a system heated from the side or from below by a uniform heat flux. Using a parallel flow approximation, a model was developed that could predict the flow and heat transfer in terms of different physical properties of the two fluid layers. Also, the critical Rayleigh number for the onset of motion was obtained explicitly in the case of a system heated from the bottom. The dynamical role of the interface viscosity has been investigated experimentally by Cardin and al. [8].

Recently, The case of a cavity divided by a partition (plywood or concrete) is numerically examined. The left side was filled with water and right one was filled with air [10]. A numerical study to examine laminar natural convection flow fields and temperature distribution in a partially divided enclosure. The vertical walls maintained at constant temperatures [11]. The nonlinear thermocapillary and buoyant-thermocapillary flows in an asymmetric three-layer system, filling a closed cavity and subjected to a temperature gradient directed along the interfaces, are investigated [12]. Analytical solution for thermal problems in hydrodynamics developed laminar flow of two immiscible liquids in a circular pipe [13]. Numerical simulation of natural convection in a container filled with two immiscible fluids is investigated [14]. natural convection heat transfer in a two-fluid stratified system with uniform internal heat sources is investigated theoretically [15]. Flow in a two-immiscible liquids layer with or without free surface was investigated. Only with the interfacial tension gradient, a couple of counterclockwise vortices were observed [16]. An experimental study of natural convection in fluid-superposed porous layers heated locally from below is reported. The effects of the heater-to-cavity length ratio and the porous layer-to-cavity height ratio on the overall heat transfer coefficients are reported. Temperature profiles in the domain reveal a plume like flow with a single pair of circulating cells [18]. Stability analyses of thermal and/or solutal natural convection in a configuration composed by a fluid layer overlying a homogeneous porous medium have been performed using different modeling approaches, especially for the treatment of the interfacial region. Comparisons between the one-domain approach and the two-domain formulation have shown important discrepancies of the marginal stability curves [19]. Numerical method is applied to predict the velocity and temperature distribution in two immiscible liquid layers with undeformable interface for a wide range of Marangoni numbers. The laminar-turbulent transition is demonstrated by obtaining the turbulence features at high interfacial temperature gradient which is characterized by high Marangoni number [20]. The steady laminar thermocapillary convection in the thin annular two superposed horizontal liquid layers with one free surface, one liquid/liquid interface subjected to a radial temperature gradient was investigated using asymptotical analysis. The asymptotic results indicate that the expressions of velocity and temperature fields in the core region are valid in the limit of the small aspect ratio [21].

The main purpose of the present paper is to develop an analytical model and numerical solution for the natural convection in a cavity contains two superposed immiscible fluids layer heated from below and laterally. Such a condition, which may be encountered in practical situations, has not been quite enough studied. Therefore, this problem is presented in this paper with more attention. Indeed, the detection of multiple solutions in this situation and instabilities that can be generated is paramount. Particular attention is paid to the effects of interfacial tension, the temperature dependence of the fluid–fluid interface and the shear stress on the structure of the resulting flow, the temperature field, and the transfer of heat.

2. Problem position

The geometry of the physical system considered here is shown in Fig. 1. We consider a two-dimensional horizontal enclosure of height H' and width L', filled with two superposed homogeneous non-miscible fluids. A uniform heat flux per unit area (q') is applied on the two horizontal walls while the right and left vertical boundaries are subject to horizontal uniform heat fluxes (aq'). It is assumed that the flow is incompressible and the binary fluid is

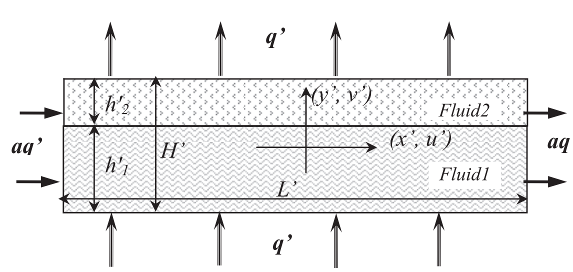

Fig. 1. Coordinate systems and configuration of the two layer physical problem.

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