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Triple convective-diffusion boundary layer along a vertical flat plate in a porous medium saturated by a water-based nanofluid

Z.H. Khan [a, d], J.R. Culham [b], W.A. Khan [b, *], I. Pop [c]

[a] School of Mathematical Sciences, Peking University, Beijing 100871, PR China
[b] Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
[c] Department of Mathematics, Babeş-Bolyai University, 400084 Cluj-Napoca, Romania
[d] Department of Mathematics, University of Malakand, Dir (Lower), Khyber Pakhtunkhwa, Pakistan

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ABSTRACT

In this article, we investigate steady triple convective-diffusive boundary layer free convection flow past a vertical flat plate embedded in a porous medium filled by a water-based nanofluid and two salts. The plate is assumed to be convectively cooled by a surrounding fluid. It is assumed that there is no nanoparticle flux at the surface and the effect of thermophoresis is taken in to account in the boundary condition. The effects of Brownian motion and thermophoresis parameters are also introduced through a Buongiorno model in the governing equations. The self-similar solutions are obtained numerically using an implicit finite difference method. The effects of the buoyancy ratio, regular Lewis numbers and modified Dufour parameters of both salts and nanofluid parameters on the flow and heat transfer are investigated. It is found that the heat transfer rate increases as we include nanoparticles and salts. Furthermore, it is also shown that the Brownian motion has negligible effects on reduced Nusselt number.

1. Introduction

In medical sciences and engineering, the use of nanofluids in enhancing convective heat transfer is increasing day by day. Nanofluids are the liquid suspension containing nano-sized particles of various materials, such as oxide ceramics, nitride ceramics, carbide ceramics, metals, semiconductors, carbon nanotubes and nanocomposites. Convective flow in porous media saturated with nanofluids have been extensively studied by many investigators owing to its several applications in engineering such as post accidental heat removal in nuclear reactors, solar collectors, and heat exchangers. Steady free convection about a vertical flat plate embedded in a saturated porous medium without any nanofluid has been investigated by a large number of researchers [1–15] among others. They considered different thermal boundary conditions and obtained analytical and numerical solutions. They investigated the effects of suction/injection, internal heat generation, radiation on the dimensionless velocity, temperature, local skin friction and the local rate of heat transfer with uniform and variable thermophysical properties.

Natural convective boundary layer flow in a porous medium saturated with a nanofluid over a vertical plate under different thermal boundary conditions has been considered by a number of researchers. For example, Kuznetsov and Nield [16] studied analytically the natural convective boundary-layer flow of a nanofluid past a vertical plate. Later on, Kuznetsov and Nield [17] studied the double-diffusive natural convective boundary-layer flow of a nanofluid past a vertical plate. They employed a Buongiorno model and included regular diffusion and cross-diffusion terms in the energy equation. A similarity solution was presented. Numerical calculations were performed in order to obtain correlation formulas giving the reduced Nusselt number as a function of the various relevant parameters.

Khan and Aziz [18–20] used the nanofluid mathematical model proposed by Buongiorno to study double-diffusive natural convection from a vertical plate under different thermal boundary conditions. A porous medium saturated with a binary base fluid containing nanoparticles was considered, the porous medium being described by the Darcy model. They investigated the effects of various parameters on the flow, heat and mass transfer characteristics. Also, correlations for the Nusselt and Sherwood numbers

* Corresponding author.
E-mail addresses: wkhan_2000@yahoo.com, wkhan1956@hotmail.com (W.A. Khan).

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Nomenclature

A	actual value of each graph
C	solutal concentration of nanofluid
C_1	solutal concentration of salt 1
C_2	solutal concentration of salt 2
D	default value
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
D_{CT}	Soret diffusivity
D_{TC}	Dufour diffusivity
D_{Sm}	solutal diffusivity of porous medium
g	acceleration due to gravity
k	effective thermal conductivity of nanofluid
k_p	thermal conductivity of nanoparticles
k_f	thermal conductivity of base fluid
K	permeability of the porous medium
Ld	Dufour-solutal Lewis number
Le	regular Lewis number
Ln	nanofluid Lewis number
Nb	Brownian motion parameter
Nc	regular double-diffusive buoyancy parameter
Nd	modified Dufour parameter
Nr	buoyancy ratio
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Nur	reduced Nusselt number
Pr	Prandtl number
Ra_x	local Rayleigh number
Sh_x^1	local solutal Sherwood number for salt 1
Sh_x^2	local solutal Sherwood number for salt 2
Sh_x^n	local nanoparticle Sherwood number
Shr^1	reduced solutal Sherwood number for salt 1
Shr^2	reduced solutal Sherwood number for salt 2
Shr^n	reduced nanoparticle Sherwood number
T	local fluid temperature
T_∞	ambient temperature
$u,\ v$	velocity components along x and y directions
x	coordinate along the plate
y	coordinate normal to the plate.

Greek symbols

α_m	thermal diffusivity of porous medium
β_T	volumetric thermal expansion coefficient of the fluid
β_C	volumetric solutal expansion coefficient of the fluid
χ	dimensionless solutal concentration
ε	porosity of the medium
ϕ	rescaled nanoparticle volume fraction
ϕ_w	nanoparticle volume fraction at the wall
ϕ_∞	ambient nanoparticle volume fraction
η	similarity variable
μ	absolute viscosity of the base fluid
ν	kinematic viscosity of the fluid
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_m$	effective heat capacity of the porous medium
$(\rho c)_p$	effective heat capacity of the nanoparticle material
τ	ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid
θ	dimensionless temperature
ψ	stream function

based on regression analysis of the data were established. These linear regression models provide a highly accurate (with a maximum standard error of 0.004) representation of the numerical data and can be conveniently used in engineering practice. Nield and Kuznetsov [21] revisited the Cheng–Minkowycz [1] problem for different boundary-layer flows in a porous medium saturated by nanofluids using Buongiorno's model. Nield and Kuznetsov [22–24] then extended this work to the case of double diffusion, bidisperse and tridisperse porous medium saturated with nanofluids. Recently, Nazari et al. [25] studied the effect of suction or injection on free convection boundary layer past a vertical heated plate embedded in a saturated porous medium when the porous medium is in thermal non-equilibrium. They assumed a linear temperature distribution and calculated the reduced Nusselt numbers for both the solid and fluid phases. Further, Gorla and Chamkha [26] presented a boundary layer flow analysis for natural convection past a nonisothermal vertical plate in a porous medium saturated with a nanofluid. They investigated the effects of controlling parameters on friction factor, surface heat and mass transfer rates.

Recently, Rionero [27] presented the idea of more than one chemical dissolved in fluid mixtures for describing natural phenomena like contaminant transport, underground water flow and acid rain effects. Later on, Khan and Pop [28] extended the idea of Rinoreo [27] to the case of a triple convective-diffusive fluid mixture past a vertical plate in a saturated medium. In this study, the same idea is extended to triple diffusion along a vertical plate in porous media saturated with nanofluids. It is important to note that the results of this study are completely new and useful in practical applications.

2. Basic equations

We consider triple-diffusive convective boundary layer flow along a vertical impermeable flat plate embedded in a porous media saturated with a water-based nanofluid. The problem is assumed to be two-dimensional with the $x-$ axis aligned vertically upwards and $y-$ axis normal to it. It is assumed that the base fluid of the nanofluid is water containing two different components of salt having different properties. It is also assumed that the temperature of the plate is T_f, while that of the ambient fluid is T_∞, where $T_f > T_\infty$ (heated plate). The effects of Brownian motion and thermophoresis are under consideration, while the Darcy model is used for the porous medium. In addition, the thermal energy equations include regular diffusion and cross-diffusion terms for both components of concentration, C_1 andC_2. The standard boundary layer scale analysis gives the following governing equations for the proposed model,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\begin{aligned}\frac{\partial P}{\partial x}=-\frac{\mu}{K}u+(1-C_\infty)g\rho_{f\infty}\{\beta_T(T-T_\infty)+\beta_{C_1}(C_1-C_\infty)\\+\beta_{C_2}(C_2-C_\infty)\}-\left(\rho_p-\rho_{f\infty}\right)g(C-C_\infty),\end{aligned} \tag{2}$$

$$\frac{\partial P}{\partial y}=-\frac{\mu}{K}\mathrm{v}, \tag{3}$$