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An analytical solution for convective heat transfer of viscoelastic flows in rotating curved pipes

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A B S T R A C T

This research reports an analytical solution for flow and convective heat transfer of an Oldroyd-B viscoelastic fluid in rotating curved pipes with viscous dissipation effects. In order to solve the momentum and energy conservation equations, a perturbation method is employed. Fully-developed hydrodynamic and thermal boundary conditions are considered with a constant heat flux imposed at the pipe wall. A physical solution based on the *H2* boundary condition is presented and it is shown that the results of the present study are more physically realistic than those reported in previous studies utilizing only the *H1* assumption. The computations have shown that due to the large gradient of velocity field and rheological properties of viscoelastic fluids, the internal heat generated by viscous dissipation exerts a significant role on both flow and heat transfer characteristics in the present regime. Furthermore, the interesting phenomenon of dissipation of mechanical energy is also investigated. Therefore, the current study emphasizes the substantial effects of viscous dissipation on the heat transfer in non-symmetric temperature distributions. The present analysis evaluates in detail the collective effects of Coriolis force, inertia forces, elastic force and dissipation effects, via the Rossby, Reynolds, Weissenberg and Brinkman numbers, respectively. An increase in Rossby number for the co-rotation case is observed to displace the minimum temperature location toward the outer side of the pipe and additionally enhances the sensitivity of the flow to Reynolds, Weissenberg, Prandtl numbers, viscosity ratios and curvature ratios. Visualization of the main and secondary flows is also presented. The study is relevant to thermal polymeric flow processing.

1. Introduction

Simulation of fluid flow through curved pipes is a fundamental problem in industry and has attracted the attention of many investigators due to its diverse range of applications. These include but are not restricted to biomechanics [1–5] and chemical engineering [6,7]. Dean [8,9] initiated theoretical studies of viscous flow in curved tubes, and employing the loose-coiling approximation and neglecting all curvature effects with the exception of centripetal acceleration, simplified the Navier–Stokes equations. He derived an analytical solution based on a perturbation method for the case of a stationary curved pipe and highlighted key features of the flow such as Taylor–Görtler secondary flows, corroborating the earlier empirical investigation of Eustic [10]. Subsequently, Topakoghlu [11] derived perturbation solutions also for Newtonian viscous flows in both curved pipe circular and annular geometries. A seminal review of progress up to the early 1980s was made by Berger [12] and later by Guan and Martonen [13] and Naphon and Wongwises [14].

Heat transfer in laminar flow is also an important phenomenon in modern processing industries and curved tube internal convection has received significant attention. Thermal characteristics computed for such flows facilitate the optimized design, construction and operation of heating and cooling systems, in which pipe components rotate. The centrifugal force arising from the rotation of pipe contributes to the generation of secondary flows. The interaction of this secondary flow with boundaries makes the heat transfer a complex issue, deviating markedly in nature from stationary curved pipe heat transfer. Early studies of transport phenomena in rotating curved pipes were restricted to Newtonian flows. Ito and Motai [15] conducted a seminal investigation focusing on the co-rotating and counter-rotating curved duct for a

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Newtonian fluid and established the reversal in secondary flow direction theoretically for low Dean numbers. Menon [16] confirmed this reversed secondary flow for high Dean Numbers. Ishigaky [17] presented numerical solutions for such flows and introduced the ratio of Coriolis force to centrifugal force as a new parameter *F* and identified that when F is about −1 (corresponding to the case where the Coriolis and centrifugal forces possess the same magnitude in opposite direction) the secondary flow arising from centrifugal and Coriolis forces coexists and the friction factor attains the same value as for the stationary straight conduit. Zhang et al. [18] also developed perturbation solutions for Newtonian rotating curved conduit flow.

In recent decades, non-Newtonian fluid flows in curved and rotating ducts have stimulated substantial interest, owing to applications in cooling systems for conductors of electric generators, separation processes, rotating heat exchangers and turbomachinery which are working with non-Newtonian fluids. Generally in most investigations, heat transfer has been shown to have a major impact on production rates and characteristics of manufactured materials. Combination of high viscosities of polymeric materials and large deformation rates results in transformation of a large amount of mechanical energy into heat and therefore elevation of fluid temperature. In extruders, this phenomenon is so significant that is used to enhance the melting process in industry and therefore should be incorporated in mathematical modeling of the process.

In viscoelastic fluids, besides the centrifugal force owing to the curvature of conduits, the elastic force also contributes in inducing Taylor–Görtler vortices. In this situation, near the outer side of the curvature, the first normal stress differences result in a strong hoop stress which intensifies secondary flows and subsequently modifies the associated thermal convection fields. Jitchote and Robertson [19] examined the effect of these normal stress differences in creeping flow (neglect inertial effects) and showed that for such Stokesian flows, a variation in normal stress differences strongly influences the intensity of secondary flows. Fan et al. [20] employed Oldroyd-B viscoelastic and UCM (Upper Convected Maxwell)-N_2 models in computational simulations of inertial flow in curved pipes, aimed at elucidating the effect of normal stress differences on secondary flow intensity and flow resistance ratio. They showed that increasing the first normal stress differences intensifies the secondary flows while enhancing the negative second normal stress differences manifests in the opposite effect. Important verification of the influence of these normal stresses on thermofluid transport in curved pipes has been communicated very recently by Norouzi et al. [21]. Previously, Bowen et al. [22] used the perturbation method to investigate the creeping flow of upper convective Maxwell and second order fluids. They showed that the time constants of these two models (i.e. relaxation and retardation time constants) have the reverse effect on flow rates in creeping regimes. Subsequently Robertson and Muller [23] extended the model to the more general case by considering Oldroyd-B inertial flow through circular and annular cross-section curved pipes and evaluating the effects of Weissenberg number and Reynolds number. Comprehensive experimental research [24,25] on the effects of elasticity in inducing secondary flows has also been presented, these studies establishing in detail that the intensity of this secondary flow is also affected by dimensionless curvature and first normal stress differences as well as centrifugal force. Hartnett and Kostic [26] rigorously analyzed non-Newtonian heat transfer in straight rectangular cross-section ducts, highlighting that heat transfer for viscoelastic flows in straight ducts may be greater than those for Newtonian fluid, this pattern being attributable to the secondary flow generated from elastic effects.

It is interesting to mention that the vast majority of heat transfer investigations concerning viscoelastic fluids have omitted viscous dissipation effects. Generally it is assumed that internal energy is function of fluid temperature only. However it should be noted that this assumption is no longer accurate for viscoelastic materials. Braun and Friedrich [27] and Ko and Lodge [28] have conducted some interesting studies in this field. Peters and Baaijens [29] have communicated an interesting investigation in this regard to determine which part of the supplied mechanical energy is dissipated and irreversible and which part is stored as elastic energy and can thereafter released as mechanical energy again. In this scenario, two limiting cases are considered, namely ideal elastic and viscous materials. A constitutive equation for viscoelastic materials is developed and is related to the energy equation such that mechanical energy is divided into two components in viscoelastic materials – a component which is stored as elastic energy and a component which is dissipated. Two different mechanisms for storing elastic energy in a viscoelastic material are entropy and internal energy elasticity. The first is attributed to the entropy part of the Helmholtz free energy and the second one to the internal energy part.

Of the relatively sparse studies which have been conducted in this field, several are interesting. Pinho et al. [30] have shown that in simplified Phan-Thien–Tanner viscoelastic fluids, heat which is generated via viscous dissipation has an important influence on fluid elasticity and heat transfer by imposed heating at the wall. Hashemabadi and Mirnajafizadeh [31] investigated the effects of viscous dissipation on heat transfer by utilizing the Nahme–Griffith number as the perturbation parameter. They observed that viscous dissipation may considerably alter isothermal flow characteristics and serves to generally enhance Nusselt number.

Several investigators have also addressed the mutual effects of elastic force, curvature and rotating body forces on non-Newtonian fluid flow. Chen et al. [32] analytically studied the characteristics of the Oldroyd-B flow in a rotating curved pipe. Shen et al. [33] studied the flow and heat transfer for iso-flux Oldroyd-B fluids in rotating curved pipe by implementing the *H1* boundary condition. This boundary condition unlike the *H2* boundary condition uses an additional assumption for iso-flux scenario that considers peripherally constant temperature in each cross section varying along the main direction of pipe (temperature of wall is function of the main direction). It is pertinent to emphasize that this assumption is developed to overcome the problem of Dirichlet boundary condition. As it is generally known, boundary conditions for heat transfer problems which consider a constant imposed heat flux at the wall are represented in Neumann form. In order to obtain a unique steady solution implementing a Dirichlet boundary condition is necessary.

It is interesting to note that in the axisymmetric scenario this additional assumption fits well with physical situation and there is no difference between these two solutions. Surprisingly, as soon as the flow obtains a non-axisymmetric pattern a difference between these two solutions arises which makes this solution (*H1*) deviate from the physical situation (*H2*). Shah and London [34] in an axisymmetric scenario showed that the temperature distribution at each cross section of the wall is uniform (*H1* and *H2* solutions are the same) while in non-axisymmetric conditions such as curved pipes and non-circular conduits this assumption no longer simulates the correct physics of the iso-flux scenario. Norouzi et al. [35] did the same and computed solutions for an Oldroyd-B fluid in curved pipe convection and showed this assumption leads to deviation between these two solution (*H1* and *H2* situation) as Reynolds and Weissenberg number is growing. The problem is also studied for convective heat transfer of viscoelastic flows in non-circular ducts by Shahmardan et al. [36] and Norouzi et al. [37,38].

As mentioned before, Shen et al. [33] investigated the heat transfer in an Oldroyd-B fluid in rotating curved pipe flow,