International Journal of Thermal Sciences 90 (2015) 187–196

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Spectral collocation method for convective–radiative transfer of a moving rod with variable thermal conductivity

Ya-Song Sun [a], Jing Ma [b], Ben-Wen Li [c,*]

[a] Beijing Key Laboratory of Multiphase Flow and Heat Transfer for Low Grade Energy, North China Electric Power University, Beijing 102206, China
[b] School of Automobile, Chang'an University, Xi'an 710064, China
[c] Institute of Thermal Engineering, School of Energy and Power Engineering, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 30 April 2014
Received in revised form
13 December 2014
Accepted 14 December 2014
Available online

Keywords:
Spectral collocation method
Moving convective–radiative rod
Temperature dependent thermal conductivity
Nonlinear differential equation

ABSTRACT

In the thermal processing of continuous casting and rolling, the metal is continuously moving. The sections of the moving metal can be rod, sheet or other structural ones. Usually, the thermal conductivity of metals will vary with temperature. In this paper, the spectral collocation method (SCM) is presented and formulated to simulate the heat transfer process in a continuously moving convective-radiative rod with variable thermal conductivity. In this approach, the dimensionless temperature is approximated by Chebyshev polynomials and discretized by Chebyshev–Gauss–Lobatto collocation points. A particular algorithm is used to reduce the nonlinearity of the energy conservation equation. Compared with those available data in literature, the SCM can provide good accuracy for a wide range of parameters, such as the dimensionless thermal conductivity coefficient, the convective–conductive parameter, the radiative-conductive parameter, the Peclet number, the dimensionless convective sink temperature, and the dimensionless radiative sink temperature. Meanwhile, the SCM can provide exponential convergence rate against node for the present problem. Moreover, the effects of various aforementioned parameters on the dimensionless temperature distribution and the dimensionless tip temperature are discussed and physically interpreted.

1. Introduction

The thermal performance analysis of fin or extended surface is one of the fundamental topics in the field of heat transfer [1]. It has widely applications in engineering industries, such as cooling system of space radiators, heat radiators in automobiles, heat exchangers in power plants, etc. When doing thermal performance analysis of the fin, the assumptions of constant thermo-physical properties (such as, thermal conductivity, heat transfer coefficient and surface emissivity) can reduce the mathematical complexity of energy equation, and allow to obtain analytical solution for energy equation [2]. However, if a large temperature difference between the fin-tip and fin-base exists, variation of the thermal conductivity is very significant and should be considered as temperature-dependent [3]. Because the variable thermal conductivity will increase the nonlinearity of energy conservation equation, it is impossible to obtain the analytical solution. Therefore, one alternative way to solve energy conservation equation is by approximation or numerical method.

Aziz and Huq [4] used a perturbation method to analyze heat transfer process in the convective fin with temperature-dependent thermal conductivity as early as in 1975. Yu and Chen [5] gave rigorous formulations using a Taylor transformation, and investigated the optimal fin length of the convective-radiative rectangular straight fin with variable thermal conductivity. Chang [6] used the Adomian decomposition method (ADM) to analyze the thermal characteristics of the rectangular fin with power-law temperature-dependent heat transfer coefficient, while Arslanturk [7] extended the ADM to obtain the temperature distribution and the fin efficiency. Khani et al. [8] utilized the homotopy analysis method (HAM) to evaluate analytical approximate solutions and the efficiency of the convective fin problem with temperature-dependent thermal conductivity and heat transfer coefficient. Chowdhury et al. [9] developed the HAM to obtain the temperature distribution of the rectangular fin with power-law temperature-dependent surface heat flux, and compared with those results of homotopy perturbation method (HPM) and ADM. It was found that HPM and ADM are peculiar cases of HAM for this problem. Malekzadeh and

* Corresponding author. Tel.: +86 13664102228.
E-mail address: heatli@dlut.edu.com (B.-W. Li).

http://dx.doi.org/10.1016/j.ijthermalsci.2014.12.019
1290-0729/

Rahideh [10] applied the finite difference method (FDM) and the different quadrature method (DQM) to analyze heat transfer in a pin fin with different boundary conditions. In Ref. [11], the variational iteration method (VIM) was proposed to solve nonlinear equations arising in heat transfer. The differential transformation method (DTM) was used to investigate the fin efficiency of convective fins with variable thermal conductivity [12], the temperature distribution of convective-radiative fin with multiple nonlinearities [13], and the thermal performance of convective-radiative straight fins with various cross-sections [14]. Recently, the least square method (LSM) [15] was adopted for predicting temperature distributions in circular porous fins with different section shapes and materials.

After the year 2000, many researchers have studied heat transfer of the moving convective-radiative fin. Aziz and Khani [16] proposed the HAM with 20 terms of series to solve heat transfer in the moving fin with temperature dependent thermal conductivity and heat loses by both convection and radiation. Aziz and Lopez [17] used a numerical algorithm built into Maple 14 to investigate the thermal processing in a continuously moving rod with variable thermal conductivity and considering both convective and radiative heat losses. Torabi et al. [18] developed the DTM to solve this kind of problem, while Kanth and Kumar [19,20] adopted the Haar wavelet method (HWM). Recently, Saedodin and Barforoush [21] applied the DTM to analyze the thermal processing of moving convective-radiative plates with temperature-dependent thermal conductivity, heat transfer coefficient and surface emissivity.

In the community of computational mechanics or numerical simulations, spectral collocation method (SCM) is one of the spectral methods, which are high order numerical methods and can provide exponential node convergence rate (in other words, spectral accuracy) [22–24]. Due to the mathematical simplicity and computational efficiency, the SCM has turned out to be an efficient tool in science and engineering applications, such as computational fluid dynamics [25–28], quantum physics [29], magneto-hydrodynamics [30–33] and thermal radiation heat transfer [34–38]. To the best of our knowledge, the SCM has not been applied to analyze the heat transfer in the moving convective-radiative rod with temperature-dependent thermal conductivity.

In this research, the convective-radiative heat transfer of a continuously moving rod with temperature-dependent thermal conductivity is investigated by the SCM. In the following of this paper, the physical model and mathematical formulation are presented in Section 2. In Section 3, the accuracy and convergence rate of the SCM against node are demonstrated by available results from references. In addition, the effects of six dimensionless parameters, including the dimensionless thermal conductivity coefficient a, the convective–conductive parameter N_{cc}, the radiative–conductive parameter N_{rc}, the Peclet number Pe, the dimensionless convective sink temperature Θ_a and the dimensionless radiative sink temperature Θ_r, on the dimensionless temperature distribution and the dimensionless tip temperature in the moving rod are also analyzed also in Section 3. Finally, conclusions are summarized in Section 4.

2. Mathematic formulation

As shown in Fig. 1, we consider the thermal processing of a moving rod with temperature-dependent thermal conductivity. The shape of moving rod is defined with cross-sectional area A and perimeter P. The velocity of the moving rod is v. The hot rod emerges from a hotter environment at a constant temperature T_b to a colder temperature and loses heat by natural convection and radiation. The convective sink temperature T_c and radiative sink temperature T_r are assumed to be different, and can vary independently. The surface of the moving rod is assumed to be diffuse and gray, the surface emissivity ε and the heat transfer coefficient h are assumed to be constant. The radiative heat exchange between the rod and the tip is neglected. If a large temperature variation exists within the moving rod, the thermal conductivity of the moving rod may vary with temperature, and can be taken as [17,20]

$$\lambda = \lambda_c[1 + a'(T - T_c)] \tag{1}$$

where λ_c is the thermal conductivity at the convective sink temperature T_c, and a' is the thermal conductivity coefficient which is determined by material. For example, the thermal conductivity coefficient of aluminum is $a' = -3.9375 \times 10^{-4}\ \mathrm{K}^{-1}$ when the rod temperature decreases from 800 K to 100 K [3].

From the view of energy conservation, the steady-state energy equation of the moving rod with a constant speed and heat loses through natural convection and radiation can be expressed as [18]

$$\frac{d}{dx}\left[\lambda(T)\frac{dT}{dx}\right] - \frac{hP}{A}(T - T_c) - \frac{\varepsilon\sigma P}{A}\left(T^4 - T_r^4\right) + \rho c_P v\frac{dT}{dx} = 0 \tag{2}$$

where ρ is the density of material, c_P is the specific heat capacity at constant pressure.

There are Dirichlet boundary condition and Neumann boundary condition for the moving rod. As shown in Fig. 1, the boundary condition at base for Eq. (2) is assumed to be constant temperature T_b, likely

$$T(x = 0) = T_b \tag{3}$$

The boundary condition at tip for Eq. (2) is assumed to be adiabatic and can be written as

$$\left.\frac{dT}{dx}\right|_{x=L} = 0 \tag{4}$$

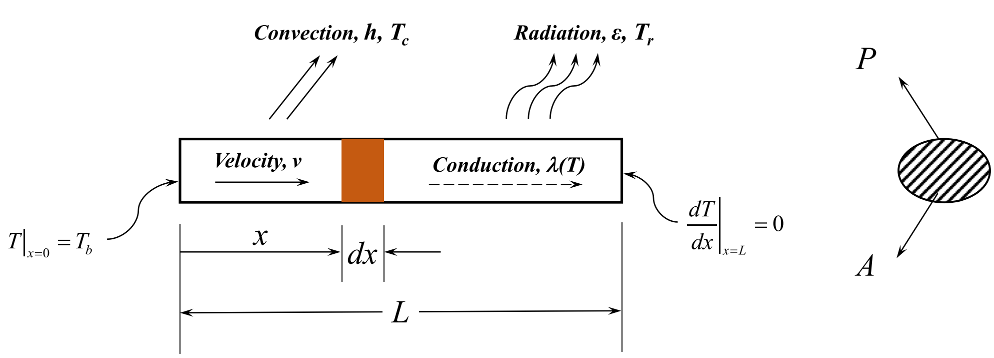

Fig. 1. Schematic diagram of convection and radiation from the surface of a moving rod.