International Journal of Thermal Sciences 86 (2014) 60–67

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Triple diffusion along a horizontal plate in a porous medium with convective boundary condition

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ARTICLE INFO

Article history:
Received 20 December 2012
Received in revised form
1 May 2014
Accepted 26 June 2014
Available online 22 July 2014

Keywords:
Triple diffusion
Natural convection
Horizontal plate
Porous medium
Nusselt number
Sherwood number

ABSTRACT

The effects of triple diffusion on a horizontal plate immersed in a porous medium are investigated subject to a convective boundary condition. A Darcy model is used to analyze two different scenarios where salts with different compositions are dissolved in a fluid-saturated porous medium. The governing partial differential equations are reduced to ordinary (similarity) differential equations that are solved numerically using an implicit finite difference method with a quasi-linearization technique. Results are compared with the available data and are found to be in excellent agreement. Results of the dimensionless velocity, temperature, salts concentrations, reduced Nusselt and Sherwood numbers are presented for both buoyancy aiding and opposing flows.

1. Introduction

Combined natural heat and mass transfer, the so-called double-diffusive or thermosolutal convective problem, has become an increasingly attractive field of study for researchers and engineers in many diversified areas. The phenomenon of combined mass and heat transfer driven by natural convection in porous media has received considerable attention due to its many relevant applications such a chemical engineering, environmental dynamics, spray drying and flash drying, cyclone evaporation, chemical and food processing plants, polymer production, the migration of moisture through the air contained in fibrous insulation and grain storage insulations the dispersion of chemical contaminants through water-saturated soil, to name but a few (Bejan and Khair [1], Khair and Bejan [2], Mahajan and Angirasa [3], etc.). Heat and mass transfer transport involving buoyancy effects caused by diffusion of heat and chemical species are found in numerous naturally occurring processes as well as multiple industrial applications. The study of these processes is useful for providing important physical understanding and subsequently improving a number of chemical technologies. A very good review of this topic can be found in the book by Nield and Bejan [4]. Also, an excellent review paper on the double-diffusive convection and finite-amplitude flows has been published by Mamou [5].

The problem of natural convection in a porous medium past a horizontal plate is a classical problem, which has been first studied by Cheng and Chang [6]. This problem has been further extended by Chang and Cheng [7], Chandrasekhara [8], Lai and Kulacki [9,10], and Merkin and Zhang [11]. In a brief communication, Poulikakos [12] explained the impact of a third diffusing component on the onset of double diffusive convection in a horizontal fluid-saturated porous layer. He performed a linear stability analysis and found that the presence of the third diffusing component with small diffusivity can significantly change the nature of the convective instabilities in the system.

Recently, Rionero [13] discussed the importance of considering secondary dissolved in fluid mixtures when describing natural phenomena (contaminant transport, underground water flow, acid rain effects, warming of the stratosphere). Consequently, this study investigates a triple convective-diffusive fluid mixture saturating a porous horizontal layer, heated from below and salted from above and below. In closed forms, conditions sufficient for (i) inhibiting the onset of convection and (ii) guaranteeing the global nonlinear

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http://dx.doi.org/10.1016/j.ijthermalsci.2014.06.035
1290-0729/

Nomenclature

C_m	concentrations
D_m	mass diffusivity of solute S_m
f	reduced stream function
$\mathbf{g}$	gravity vector
K	permeability of the porous medium
Le	Lewis number
N	buoyancy ratios parameter
Nu_x	local Nusselt number
p	pressure
Ra_y	local Rayleigh number
S_m	chemical components ("salts")
Sh_x	local Sherwood number
T	fluid temperature
$\mathbf{v}$	velocity vector
u, v	velocity components along the x and y axes
x, y	Cartesian coordinates measured normal and parallel to the plate

Greek letters

α	thermal diffusivity of the porous medium
β_T, β_1, β_2	coefficient of thermal expansion of solute S_m
χ	dimensionless concentration
ϕ	dimensionless concentration
η	similarity variable
μ	dynamic viscosity
ν	kinematic viscosity
θ	dimensionless temperature
ρ	density
ψ	stream function

Notations

$m = 1, 2$	salt identifier
w	condition at the surface
∞	free stream condition

stability of the thermal conduction solution and the absence of subcritical instabilities were obtained. Using the idea of Rionero [13], we extend the study of Cheng and Chang [6] to the case of a triple convective-diffusive fluid mixture ($m = 2$), with the aim of making a detailed comparison with the case of a double-diffusive natural convective boundary layer and at the same time investigating the interaction between the salt S_1 and S_2, respectively. As far as we know, the results of this paper appear to be completely new in the existing literature and in view of their simplicity, easily transferrable to relevant applications.

2. Basic equations

Let us first consider steady, free convection boundary layer flow past a horizontal plate embedded in a fluid-saturated porous medium where the flow is assumed to be two-dimensional. The coordinate system is oriented in such a way that x- and y-axes are in the horizontal and vertical directions, respectively. In keeping with the Boussinesq approximation, the porous medium is assumed to be homogeneous and at a thermal equilibrium in the localized domain. The assumption of a local thermal equilibrium allows for consideration of a one-equation model for the heat transport in a porous medium. Following Rionero [13], we also assume that two different chemical components ("salts") S_m ($m = 1$, 2) have dissolved in the fluid-saturated porous medium, which have concentrations C_m ($m = 1, 2$), respectively, and that the equation of state is

$$\rho = \rho_\infty[1 - \beta_T(T - T_\infty) + \beta_1(C_1 - C_\infty) + \beta_2(C_2 - C_\infty)] \tag{1}$$

where ρ_∞, T_∞ and C_∞ are the density, temperature and salt concentrations in the free stream, while the constants β_T, β_1 and β_2 denote the coefficient of thermal expansion and solute S_m expansion coefficients, respectively ($m = 1, 2$), which are defined by

$$\beta_T = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p, \quad \beta_1 = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial C_1}\right)_p, \quad \beta_2 = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial C_2}\right)_P \tag{2}$$

Combining Darcy's Law

$$\frac{\mu}{K}\mathbf{v} = -\nabla p + \rho\mathbf{g} \tag{3}$$

with (thermal) energy and mass balance together with the Boussinesq approximation (1), we obtain the following fundamental equations governing the isochoric motions,

$$\nabla\cdot\mathbf{v} = 0 \tag{4}$$

$$\frac{\mu}{K}\mathbf{v} = -\nabla p + \rho_\infty[1 - \beta_T(T - T_\infty) + \beta_1(C_1 - C_\infty) + \beta_2(C_2 - C_\infty)]\mathbf{g} \tag{5}$$

$$\mathbf{v}\cdot\nabla T = \alpha_m\nabla^2 T \tag{6}$$

$$\mathbf{v}\cdot\nabla C_1 = D_1\nabla^2 C_1 \tag{7}$$

$$\mathbf{v}\cdot\nabla C_2 = D_2\nabla^2 C_2 \tag{8}$$

The standard boundary layer scale analysis gives the following five equations modeling the proposed problem,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$\frac{\nu}{K}\frac{\partial u}{\partial y} = -\left(\beta_T\frac{\partial T}{\partial x} + \beta_1\frac{\partial C_1}{\partial x} + \beta_2\frac{\partial C_2}{\partial x}\right)g \tag{10}$$

Table 1
Comparison of heat and mass transfer rates $-\theta'(0)$ and $-\chi_1'(0)$ when $N_2 = 0$, $N_1 = 0$, and $Le_2 = 0$.

Le_1	Gorla and Chamkha [15]		Present results	
	$-\theta'(0)$	$-\chi_1'(0)$	$-\theta'(0)$	$-\chi_1'(0)$
1	4.30E-01	4.30E-01	0.43002	4.301E-01
10	4.30E-01	1.483679	0.43002	1.482E + 00
100	4.30E-01	4.732183	0.43002	4.727E + 00
1000	4.30E-01	14.97818	0.43002	1.497E + 01