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Hydrodynamics and thermal analysis of a mixed electromagnetohydrodynamic-pressure driven flow for Phan–Thien–Tanner fluids in a microchannel

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ABSTRACT

An analytical study of the flow and temperature fields of a viscoelastic fluid in a rectangular microchannel under the simultaneous influence of electroosmotic (EO), magnetohydrodynamic (MHD) and pressure driven forces (PD) is presented. The non-Newtonian fluid obeys the constitutive relation based on the simplified Phan-Thien-Tanner model (sPTT). The analysis is primarily motivated by the need for increasing the volumetric flow rate in a microchannel by attenuating the inevitable Joule heating effect in electrokinetic flows. The governing equations are presented in dimensionless form containing six dimensionless parameters that control the flow and temperature fields: a parameter representing the viscoelasticity of the fluid, εDe_{κ}, the ratio of the pressure to electroosmotic forces, Γ, the ratio of magnetic to electroosmotic forces, $\Omega^{*}Ha^{2}$, the ratio of the thermal resistances, Λ, the ratio of the applied electric fields, γ, and the ratio of the thickness to the length of the microchannel, β_1. We report the conditions under which it is possible to take advantage on the simultaneous application of EO, MHD and PD forces. In addition, we determine the conditions that must be met to prevent the lateral flow when EO and MHD forces are considered simultaneously [1,2]. The volumetric flow rate is observed to increase in about 40% and the maximum temperature diminishes when MHD and EO forces are present in comparison with the case of a purely EO flow.

1. Introduction

The terms Lab-on-a-Chip (LOC) and Micro Total Analysis System (μTAS) stand synonymous for devices that use fluids as a working medium and integrate a number of different functionalities on a small scale. The most important of these functionalities are sample preparation and transport, separation and biosensing/detection. Much attention has been focused recently on miniature systems for chemical and biological analysis [3]. For instance, some microfluidic systems have been fabricated on a chip to study mobilization of biological cells on-chip as the yeast, escherichia coli and erythrocyte cells due to their relevance to applications in cell and molecular biology, DNA cloning and hematology [4]. Also, microfluidic devices based on the Polymerase Chain Reaction (PCR) chips are used in the amplification of DNA and for separating species through capillary electrophoresis [5–7]. The task of detecting microbes [8,9] or viruses [10] in blood samples is common in these microfluidic systems. Knowledge of the viscosity of biofluids such as blood, blood plasma, amniotic and synovial fluid has a significant role in diagnostic, prognostic and preventative medicine [11]. The fluid volumes in the above mentioned devices are often pumped, controlled or manipulated during certain specific operations, using sample volumes ranging from hundreds of picoliters to hundreds of microliters. In this sense, many microfluidic applications would benefit from an on-chip active pump with size comparable to the small volume of fluid to be pumped, i.e. an integrated micropump. However, the moving parts in displacement micropumps make the fabrication and operation delicate [12]. Therefore, the requeriment of an integrated micropump with no moving parts can be fulfilled by using electrokinetic and magnetohydrodynamic effects [6].

The electroosmosis is one of the basic electrokinetic phenomena, which refers to liquid flow induced by an applied external electric field along electrostatically charged surfaces [13]. In the treatment of biological fluids with non-Newtonian behavior, Afonso et al. [14] derived an analytical solution for mixed

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Nomenclature

a	microchannel half-hight, m
C_p	specific heat, J kg^{-1} K^{-1}
$\mathbf{B}$	magnetic field strength, Vs m^{-2}
$\mathbf{b}$	body force vector, N m^{-3}
B_y	magnetic field in the y-direction, Vs m^{-2}
$\mathbf{E}$	electric field strength, V m^{-1}
e	elementary charge, C
E_x	electric field in the x-direction, V m^{-1}
E_z	electric field in the z-direction, V m^{-1}
Ha	Hartmann number
h	convective heat transfer coefficient, W m^{-2} K^{-1}
h_{eq}	global heat transfer coefficient, W m^{-2} K^{-1}
$\mathbf{J}$	electric current density, A m^{-2}
k	thermal conductivity of the fluid, W m^{-1} K^{-1}
k_w	thermal conductivity of the microchannel wall, W m^{-1} K^{-1}
k_B	Boltzmann constant, J K^{-1}
L	microchannel length, m
n_0	ionic number concentration, m^{-3}
p	pressure, N m^{-2}
$\bar{p}$	dimensionless pressure
Pe	Peclet number
$\dot{\mathbf{q}}$	heat source, W m^{-3}
Re	Reynolds number
T	temperature, K
t	time, s
T_0	reference temperature, K
u,v,w	velocities in x-, y- and z-directions, ms^{-1}
$\bar{u},\bar{v},\bar{w}$	dimensionless velocities in $\bar{x}$-, $\bar{y}$- and $\bar{z}$-directions
u_{HS}	Helmholtz-Smoluchowski velocity, m s^{-1}
u_{MHD}	characteristic velocity of a magnetohydrodynamic flow, m s^{-1}
u_p	characteristic velocity of a pressure driven flow, m s^{-1}
$\mathbf{v}$	velocity vector, m s^{-1}
W	microchannel width, m
(x,y,z)	Cartesian coordinates
$\bar{x},\bar{y},\bar{z}$	dimensionless Cartesian coordinates
Y	stress coefficient function
z	absolute value of the valence for a (z:z) electrolyte

Greek symbols

β_1	ratio of the thickness to the length of the microchannel
β_2	ratio of the length to the width of the microchannel
Γ	ratio of the pressure to electroosmotic forces
γ	ratio of the applied electric fields
$\dot{\gamma}$	rate-of-strain tensor, s^{-1}
ΔT	temperature change
ε	dielectric permittivity, C V^{-1} m^{-1}
ε	PTT parameter
εDe_κ^2	viscoelastic parameter
ζ	zeta potential, V
η_0	polymer viscosity coefficient, Nm^{-2} s
θ	dimensionless temperature
κ^{-1}	Debye length, m
$\bar{\kappa}$	electrokinetic parameter
Λ	ratio of the thermal resistances
λ	relaxation time, s
ρ	density of the fluid, kg m^{-3}
ρ_e	electric charge density, C m^{-3}
σ_e	electrical conductivity, S m^{-1}
$\boldsymbol{\tau}$	stress tensor, Nm^{-2}
$\boldsymbol{\tau}_{kk}$	trace of the extra-stress tensor, Nm^{-2}
τ	stress, Nm^{-2}
Φ	total electric potential, V
$\tilde{\Phi}$	dimensionless parameter defined in Eq. (22)
ϕ	external applied electrical potential in the x-direction, V
φ	external applied electrical potential in the z-direction, V
Ψ	dimensionless parameter defined in Eq. (22)
ψ	electric potential within of the EDL, V
Ω^*	dimensionless parameter defined in Eq. (22)
$\Omega^* Ha^2$	ratio of magnetic to electroosmotic forces

electroomotic and pressure driven flows of viscoelastic fluids in microchannels, using the simplified Phan–Thien–Tanner model, which can model fluids such as blood, saliva and synovial fluids. In this later case and considering the obtained analytical solution, there is an extra term in the velocity profile that combines simultaneously pressure and electrical forces. This last term is absent for the newtonian case. Other studies concerning complex fluid flow of purely electroosmotic flows in microchannels [15–19] and the combined effect of electroosmotic and pressure driving forces [20,21] with complex fluids have been developed already.

Additionally to electroosmotic micropumps, the use of electromagnetic effects can be used for the applications before mentioned. In this manner, the study of flow of electrically conducting liquids in the presence of electric and magnetic fields, is called magnetohydrodynamic pumping [22]. The simple fabrication, the absence of moving parts, low voltage of operation and the possibility to achieve relatively high flow rates, being able to produce forward and reverse flows [23], make the MHD micropumps an option for chemical, medical and biological applications in microfluidic systems [24–27].

On the other hand, the rise of the temperature generated by the Joule heating in electroosmotic flows is a detrimental condition for the samples considered in the microfluidic devices as denaturation of proteins or nucleic acids [28], sample band dispersion or peak broadening, low column separation efficiency, reduction of analysis resolution, decomposition of thermally labile samples and even the formation of vapor bubbles [29]. Recently, some thermal analyses have been developed for electroosmotic flows using constitutive equations for viscoelastic [30–32], Carreu [33] and power law fluids [34], studying the effects of the inevitable Joule heating when an electric field is imposed.

In the previous paragraph, the parameters that facilitate the control the Joule heating were identified and are related to the heat produced by the applied electric field with imposed thermal boundary conditions. In this sense, the Joule heating is always present in micropumps where the fluid is driven by Lorenz forces, as result of the interaction of electric and magnetic fields. In this direction, Chakraborty and Paul [35] developed a mathematical model to study the combined effect of electromagnetohydrodynamic forces for controlling the flow of Newtonian fluids through rectangular microchannels. It is revealed that, with the aid of a relatively low-magnitude magnetic field, a substantial augmentation in the volumetric flow rates can be achieved. However, due to practical constraints, the applied electrical field needs to be restrained within stringent limits, in order to ensure that any appreciable degradation of samples appears, as a consequence of the associated Joule heating effects. Additionally, Chakraborty et al. [36] analyzed the heat transfer characteristics associated with thermally fully developed combined electromagnetohydrodynamic flows of Newtonian fluids through narrow