International Journal of Thermal Sciences 86 (2014) 365–373

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Determination of the equivalent thermal conductivity of complex material systems with large-scale heterogeneities

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ARTICLE INFO

Article history:
Received 10 March 2014
Received in revised form
28 July 2014
Accepted 28 July 2014
Available online 24 August 2014

Keywords:
Heterogeneous systems
Equivalent thermal conductivity
Laboratory measurement
3D computational modeling

ABSTRACT

A method for determination of the equivalent thermal conductivity of heterogeneous systems with large representative elementary volumes is proposed. The laboratory experiment is based on the guarded hot plate principle but the heat transfer in the designed setup is generally 3D. Therefore, 3D computational modeling is applied for the analysis of heat transport in the system. The measuring procedure begins with the calibration which is performed using a material of the known thermal conductivity and typical specimen dimensions. Then, the experiment is carried out with the investigated specimen and the steady-state values of temperatures and heat fluxes in the characteristic positions are measured. The equivalent thermal conductivity is determined in an iterative procedure, utilizing the results of the laboratory experiment as input data of the computational model. The application of the proposed method is illustrated on an example of two types of advanced hollow clay bricks. The uncertainty analysis including both the standard uncertainties of types A and B and sensitivity-aimed calculations shows that the combined standard uncertainty of the equivalent thermal conductivity is 10% which can be considered satisfactory for this kind of experiment. The main advantages of the proposed method can be seen in its simplicity and cost effectiveness, together with an acceptable accuracy. This makes good prerequisites for its successful application in future experiments.

1. Introduction

Thermal conductivity of homogeneous solids can be measured by a variety of techniques utilizing different mathematical and physical principles. The steady-state methods are often considered as reference methods. A possibility to determine temperature fields and heat fluxes in an easy and precise way belongs to the main arguments for their application. On the other hand, the long measuring times induce problems with heat loss, which may cause systematic errors. The measured specimens usually have plate, spherical or cylindrical shape. As for the practical experimental setups, the guarded hot plate arrangement [1,2] is the most frequently used. The main advantage of the transient methods is shorter measurement time, in a comparison with standard steady-state methods. However, their accuracy and reproducibility is often arguable. Among the transient methods, the hot wire method [3], hot bridge method [4], hot ball method [5], step-wise method [6], pulse method [7], flash method [8], hot disk method [9], or hot strip method [10] are the most popular.

Heat transport in porous materials is a more complex phenomenon than in homogeneous solids. The presence of pores in the solid matrix is the first problem to deal with. If they are filled with air only, the convection mode of heat transport might be important, in addition to the conduction mode which governs the transport processes in the matrix. The radiation mode should also be considered because in the presence of temperature gradient the pore walls act as sources of radiation. The pore space of most porous materials is open so that mass transfer between the interior pore system and surroundings can occur. The pores often contain certain amount of water, and in that case the heat transport is combined with water transport. Apparently, such a complex heat transfer problem cannot be described by the common Fourier's concept of heat conduction with a sufficient accuracy. A part of investigators tries to solve this problem using the adjustments in the calibration and/or data interpretation of some commonly used methods, for instance the hot disk method [11], hot bridge method [12], or the parallel-plate method [13]. Another group of scientists prefers an application of various treatments based on the effective

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http://dx.doi.org/10.1016/j.ijthermalsci.2014.07.020
1290-0729/

Nomenclature

A_{pl} [m^2]	surface of heating and cooling plates
A_{node} [m^2]	nodal surface area
A [m^2]	area of the base of the specimen
α [W m^{-2} K^{-1}]	heat transfer coefficient
d [m]	average thickness of the specimen
δ [%]	maximal admissible difference between the corrected heat fluxes on the heating and cooling plates
ε [W m^{-1} K^{-1}]	maximal admissible difference between the thermal conductivity values during the iteration process
λ [W m^{-1} K^{-1}]	thermal conductivity
n	number of measurements
s	standard deviation
q_{c} [W m^{-2}]	absolute value of the heat flux on the interface between the cooling plate and the specimen
q_{h} [W m^{-2}]	absolute value of the heat flux on the interface between the heating plate and the specimen
$q_{\mathrm{c,c}}$ [W m^{-2}]	absolute value of the corrected heat flux on the cooling plate
$q_{\mathrm{h,c}}$ [W m^{-2}]	absolute value of the corrected heat flux on the heating plate
q [W m^{-2}]	heat flux through the specimen (ideal heat flux in absence of heat gains and losses)
q_{gain} [W m^{-2}]	absolute value of the heat flux of gains
q_{loss} [W m^{-2}]	absolute value of the heat flux of losses
Q_{gain} [W]	absolute value of the heat power of gains
Q_{loss} [W]	absolute value of the heat power of losses
r	resolution of an analog measuring instrument
T_{a} [K]	temperature of the surrounding air
T_{c} [K]	temperature of the cooling plate
T_{h} [K]	temperature of the heating plate
ΔT [K]	positive temperature difference between the heating and cooling plates
T_{surf} [K]	calculated temperature in the surface nodes
u_{A}	uncertainty of type A
u_{B}	uncertainty of type B
u_{C}	combined standard uncertainty

media theory, such as an extension of the Maxwell model [14], a modification of the Christensen model [15], a unification of several basic homogenization models [16], or a generalization of the Bruggeman equation involving various distribution functions [17]. In both cases, the obtained heat transport parameter is mostly called the effective thermal conductivity, in order to emphasize its difference from the original Fourier's definition.

Determination of heat transport parameters in materials or their multi-layered systems with large-scale heterogeneities is an even more challenging task. The multi-layered thermal insulation panels with interface resistances and/or air layers, the building elements with complex internal geometry such as the advanced types of hollow bricks, large concrete blocks, and even the old-fashioned brick masonry or autoclaved aerated concrete masonry can serve as characteristic examples of such systems. Apparently, for the specimens with heterogeneities on a scale of several centimeters or tens of centimeters which are necessary for a proper investigation of such complex systems the application of most methods suitable for determination of thermal conductivity or effective thermal conductivity is not feasible. Therefore, alternative experimental or computational treatments are being sought to determine the equivalent thermal conductivity of such systems. A straightforward experimental solution to this problem consists in an application of large-scale facilities [18,19]. However, the high demands on labor and financial means necessary in this kind of experiment can present a serious problem for many laboratories. In addition, the measurement accuracy of heat fluxes, in particular, can be lower than in the laboratory conditions. A utilization of a semi-scale experiment [20,21] may be considered as a compromise with respect to the cost of the experiment but the possible uncertainties at the heat flux measurement remain basically the same as in the large-scale methods. The simplest computational option is using the nested homogenization techniques [22]. Their application is fast but they need a special calibration which is essential for their reliability. A detailed finite element solution of the whole complex material system [23,24] is another possibility how the equivalent thermal conductivity can be determined by a computational technique but without a detailed experimental calibration the uncertainties particularly in modeling the radiation heat transfer may be rather high.

In this paper, we use for the determination of equivalent thermal conductivity of heterogeneous systems with large representative elementary volumes a relatively simple laboratory experiment which is based on the guarded hot plate principle. Some preliminary ideas on the modifications of the guarded hot plate method were introduced in Ref. [25] where the adopted computational model was two dimensional. However, the heat transfer in the proposed experimental setup is generally 3D. The heat gains/losses are too high to meet the basic criteria of a common 1D guarded hot plate arrangement which means, a one dimensional model of heat transfer (steady-state, periodic, Sturm–Liouville, transient model) summarized, e.g., by Barouh and Mikhailov [26] cannot be applied. A 2D assumption may not be sufficient either. Therefore, as an extension to the work presented in Ref. [25], 3D computational modeling is applied for the determination of heat gains/losses of the laboratory system and for the identification of equivalent thermal conductivity. The main advantages of the proposed method can be seen in its simplicity and cost effectiveness, together with an acceptable accuracy which is, contrary to the previous method presented in Ref. [25], verified by a thorough uncertainty analysis. It is the combination of experimental and computational treatments which provides the method with an added value in a comparison with some other methods.

2. Method for determination of the equivalent thermal conductivity of heterogeneous material specimens

2.1. Experimental setup

The experimental setup is derived from the guarded hot plate arrangement [27]. The main principle consists in the creation of a steady-state temperature field in the investigated specimen. This is achieved using heating and cooling conductive plates. After the steady state is reached, thermal conductivity of the investigated specimen can be calculated according to

$$\lambda = \frac{q \cdot d}{\Delta T} \tag{1}$$

where λ is the thermal conductivity of investigated specimen [W m^{-1} K^{-1}], q denotes the absolute value of the heat flux through the specimen [W m^{-2}], d is the average thickness of the specimen [m] and ΔT is the positive temperature difference between the heating and cooling plates [K].