International Journal of Thermal Sciences 89 (2015) 164–173

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Competition between stationary and oscillatory viscoelastic thermocapillary convection of a film coating a thick wall

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A R T I C L E I N F O

Article history:
Received 22 May 2014
Received in revised form
31 October 2014
Accepted 5 November 2014
Available online

Keywords:
Thin liquid film
Thermocapillarity
Marangoni convection
Viscoelasticity
Thick wall
Codimension-two points

A B S T R A C T

In this paper new results on linear viscoelastic thermal Marangoni convection are presented. The constitutive equation assumed is that of the Maxwell viscoelastic fluid. The competition between stationary and oscillatory convection is shown by means of plots of codimension-two points where the corresponding critical Marangoni numbers are the same. The variation of these points is investigated in a wide range of magnitudes of the thickness and thermal conductivity of the wall. Also, a discussion is given about the dependence they have on the Biot number of the fluid-atmosphere interface. Besides, it is shown how the range of the viscoelastic relaxation time corresponding to this points is modified by the Prandtl number.

1. Introduction

Thin liquid films stability has important industrial applications. The problem of surface coating is one of them. The finishing of the coating is intimately related with the thermal Marangoni stability. The fractures found after solidification of the layer are strongly related with the Marangoni convection cells. The phenomenon has been investigated for Newtonian fluids since many years ago. Pearson [1] investigates the stationary stability of a thin layer with flat free surface considering different thermal boundary conditions. Scriven and Sternling [2] investigates for the first time the effect of free surface deformability. Takashima [3] considers the stationary free surface deformation of the layer and Takashima [4] includes the time dependence of the problem taking into account for the first time the effects of gravity in both papers. Mctaggart [5] studies the double diffusive problem of Marangoni convection when the free surface is flat. The Marangoni convection is investigated from a boundary layer point of view by Christopher and Wang [6]. Emphasis is put on the influence the Prandtl number has on heat transfer. Two free deformable surfaces can be present as in Ref. [7]. Convection in a layer with a free deformable surface and a deformable membrane is investigated in Ref. [8]. When the temperature gradient across the layer is large it is important to take into account the temperature variation of viscosity as in Slavtchev and Ouzounov [9] and Kalitzova-Kurteva et al. [10] for stationary convection with deformable free surface and in Slavtchev et al. [11] for oscillatory convection and deformable free surface. The control of Marangoni convection is important to avoid fractures in the solidification process as is investigated by Bau [12] and Or et al. [13]. In particular, Kechil and Hashim [14] assume free surface deformation and include viscosity dependence on temperature. An application to microchannels is presented in the paper by Pendse and Esmaeeli [15] who investigate the Marangoni flow in two superposed fluids when a spatially periodic temperature is applied to the wall. It is shown that the competition between thermal and hydrodynamic effects is reflected in the flow strength when the relative thickness of the layers is varied.

In applications the liquid usually presents a non Newtonian behavior. One important property is the viscoelasticity of the fluid (see Bird et al. [22]). In natural convection the viscoelastic linear and nonlinear effects have been investigated, for example, by Martínez-Mardones and colleagues [16–20]. In particular, in Ref. [17] one of the goals is to find the codimension-two point between stationary and oscillatory instability to investigate the

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http://dx.doi.org/10.1016/j.ijthermalsci.2014.11.003

Nomenclature

a	wave number
a_c	critical wave number
Bi_s	free surface-atmosphere Biot number
Bi_w	wall Biot number
d	d_w/d_f
d_f	fluid layer thickness
d_w	wall thickness
e	shear rate tensor
H_h	heat transfer coefficient
Ma	Marangoni number
Ma_c	critical Marangoni number
$\vec{n}$	free surface normal vector
P	pressure
p	pressure perturbation
Pr	Prandtl number
T	temperature
$\overline{T}$	liquid temperature
$\overline{T}_w$	wall temperature
T_h	temperature profile
u	perturbation velocity x-component
$\vec{V}$	fluid velocity vector
v	perturbation velocity y-component
w	perturbation velocity z-component

Greek

β	temperature gradient
Γ	surface tension
η	dynamic viscosity
θ	temperature perturbation
κ	fluid thermal diffusivity
κ_w	wall thermal diffusivity
λ	adimensional relaxation time (Weissenberg number)
λ_T	relaxation time
μ	viscoelastic parameter
ν	kinematic viscosity
ρ	fluid density
σ	oscillation frequency
τ	shear stress tensor
χ	χ_f/χ_w
χ_f	fluid thermal conductivity
χ_w	wall thermal conductivity

possibility of nonlinear traveling and stationary waves. For a review of this problem see Dávalos-Orozco [21].

Viscoelasticity have been taken into account in Marangoni convection by a number of authors. Getachew and Rosenblat [23] investigate the problem of a flat free surface assuming a very good conducting wall. Their main concern is to calculate the points where the curves of criticality of stationary convection intersect those of oscillatory viscoelastic convection. These intersections are called codimension-two points (see Ref. [17]). Wilson [24] investigates the instability growth rates of viscoelastic fluids with particular interest on slightly supercritical situations. Siddheshwar et al. [25], for temperature dependent viscosity, explore the oscillatory Marangoni instability of different non Newtonian fluids, in particular, the Maxwell fluid. They also assume a variety of thermal boundary conditions.

From the point of view of the linear equations, the stationary and oscillatory Marangoni convections differ not only by the absence of the time derivative in the stationary problem, but also by the presence of the Prandtl number in the oscillatory case. Physically, in stationary Marangoni convection the fluid particles are able to describe closed trajectories. This is due to the shear flow produced by the thermal perturbations which, from the wall, reach the free surface and modify the temperature dependent surface tension. If hot particles are continuously able to reach the free surface the cellular flow can be sustained heating from the wall. In oscillatory convection, all particles move at once in trajectories due to the shear flow produced by the weakening of surface tension. Nevertheless, they are not able to complete closed trajectories when the fluid has relatively high thermal diffusivity. In non dimensional form this is determined by the Prandtl number, that is, the ratio of the mass diffusivity (kinematic viscosity) over the heat diffusivity. Under these conditions, the fluid particle cools easily and it is not able to reinforce the shear flow by the weakening of surface tension. Consequently, the strong surface tension of the cold regions of the free surface dominate and the surface shear works in the opposite direction making all the particles in the bulk to go backwards to the wall where they are heated again to repeat the same process.

Therefore, depending on the Prandtl number the Marangoni convection may be stationary or oscillatory, as will be shown presently. However, it is well known that the linear Marangoni convection of a Newtonian fluid layer with a flat free surface only can be stationary (see Ref. [26]). If the free surface of a Newtonian fluid layer is allowed to deform, thermocapillary oscillations may appear first [2,4]. However, the case of a viscoelastic fluid layer with a flat free surface is different. The new degrees of freedom of the macromolecules added to the liquid motion by means of the constitutive equations, allow oscillatory Marangoni convection to appear for a smaller temperature gradient than that of the Newtonian fluid for some magnitudes of the Prandtl number and relaxation times [23].

The effect of a thick wall in Marangoni convection is investigated by Takashima [27]. The simultaneous effect of gravity and thermocapillarity is investigated by Yang [28] including a wall with finite thickness. A temperature dependent viscosity is assumed by Char and Chen [29] in a liquid layer on a thick slab. A deformable free surface is assumed by Abidin et al. [30] in the presence of buoyancy effects. The heat generation and properties of a thick wall are considered in thermocapillary convection by Arifin and Bachok [31]. The non uniformity of the basic temperature gradient may have important consequences on the instability. This is taken into account by Shivakumara et al. [32] including a thick slab. The deformability of the free surface is assumed in a layer on a thick wall by Gangadharaiah [33].

The results of thermocapillary convection including a thick slab are more realistic than those of an infinitely good conducting wall. This effect has also been investigated in natural convection of a Maxwell viscoelastic fluid by Pérez-Reyes and Dávalos-Orozco [34]. There it is shown that for certain magnitudes of the Prandtl number a codimension-two point is found where stationary and oscillatory convection compete to be the first unstable one for a range of values of the non dimensional relaxation time (Weissenberg number). An important difference between this paper and ref. [34] for natural convection is that here the results are only focused on the codimension-two points and not on the curves of criticality. However, the curves of criticality have