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Eccentricity effect on bifurcation and dual solutions in transient natural convection in a horizontal annulus

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ABSTRACT

Bifurcation phenomena and the existence of dual solutions in natural convection in eccentric annulus are numerically investigated using the lattice Boltzmann method (LBM). Separate particle distribution functions in the LBM are used to calculate the density field, the velocity field and the thermal field. Different scenarios for the onset and evolution of pitchfork bifurcation are tracked. To validate the LBM code, the occurrence of various instabilities and bifurcation phenomena in concentric annulus is analyzed for different relevant parameters, such as the Rayleigh number *Ra*, Prandtl number *Pr* and radius ratio *R*. The numerical results obtained by LBM are found to be highly consistent with other published works. Then, bifurcation phenomena in eccentric annulus are analyzed. Isotherms, streamlines and bifurcation diagrams are presented for vertical, horizontal and diagonal eccentricities. The results show that the eccentricity is a crucial factor for the instability and existence of bifurcation of natural convection in eccentric annulus.

1. Introduction

Natural convection between horizontal isothermal eccentric cylinders has been extensively studied because of its theoretical interest and its various engineering applications such as thermal energy storage systems, cooling of electronic components and transmission cables. Kuehn and Goldstein [1,2] conducted an experimental and theoretical study of natural convection in concentric and eccentric horizontal cylindrical annuli. Their experimental data is commonly used to validate most of the recent numerical studies. Glakpe et al. [3] presented numerical solutions to two-dimensional steady laminar natural convection in annuli between concentric and vertically eccentric horizontal circular cylinders. Guj and Stella [4] reported numerical and experimental buoyancy driven flow in horizontal annulus. They studied the effect of the horizontal eccentricity and found that the average Nusselt number is nearly independent of the horizontal eccentricity. Fattahi et al. [5] simulated the natural and mixed convection heat transfer problems by lattice Boltzmann model based on double-population approach. Yuan and Li [6,7] investigated the natural convective heat transfer of water near its density maximum in an eccentric horizontal annulus. Yi et al. [8] investigated the natural convection in eccentric annulus over a wide range of Rayleigh numbers using the dissipative particle dynamics method with energy conservation (eDPD).

At low Rayleigh number (*Ra*), steady convection in a horizontal annulus is attained with the basic flow field of two symmetric crescent-shaped eddies irrespective of the values of Prandtl number (*Pr*) or radius ratio (*R*). However, for *Ra* larger than a critical value, several kinds of convective flows which are dependent on *Pr* and *R* can be developed, and the difficulty in the prediction and control of transitional buoyant flows increases with the increasing of *Ra*, due to the wide spectrum of flows potentially arising from successive bifurcations. In the last decades, a considerable amount of research effort has been dedicated to exploration of natural convection instabilities and bifurcations in horizontal concentric annuli. Those works mainly include three aspects of research, namely, experimental study, numerical simulation and theoretical analysis. Powe et al. [9] made experiments of the natural convection of air ($Pr = 0.7$) by visualizing flow patterns, and categorized the flow patterns obtained by their experiments and accumulated results by other researchers in a parameter space of (*Ra*, *R*). For the wide-gap annuli, i.e., for $R > 1.71$, the instability of the crescent-shaped convection induces an unsteady two-dimensional convection that is characterized by oscillations about the longitudinal axis

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Nomenclature

$\mathbf{e}_\alpha$	propagation velocity in the α direction in the lattice
F	Buoyancy, kg m/s^2
f	particle distribution function for velocity field
g	particle distribution function for thermal field
k	thermal conductivity, W/m K
L	distance between the centers of the two cylinders
l	characteristic length, m
Nu	Nusselt number
Nu_{ave}	average Nusselt number
Nu_i, Nu_o	Nusselt number at inner and outer cylinder
Pr	Prandtl number, ν/χ
q	heat flux, W/m^2
R	radius ratio
R_a	Rayleigh number, $g\beta(T_h - T_c)l^3/\nu\chi$
R_i, R_o	radii of the inner and outer cylinders, respectively
T	temperature, K
U	non-dimensional radial velocity
$\mathbf{u}$	macroscopic velocity vector, m/s
w	weight function

Greek symbols

β	thermal expansion coefficient (1/K)
ε	eccentricity, $\varepsilon(\xi,\varphi)$
ρ	density, kg/m^3
θ	non-dimensional temperature
ξ	dimensionless radial position
τ_ν,τ_T	relaxation time for velocity and temperature fields
ν	kinematic viscosity, m^2/s
φ	azimuthal angle
χ	thermal diffusivity, m^2/s
ζ	non-dimensional time
Δ	fraction of an intersected link of curved wall in the fluid region
ΔT	Temperature difference, $T_h - T_c$
$\Delta\zeta$	non-dimensional time step

Subscripts

ave	average
eq	equilibrium distribution
i, o	inner and outer cylinder of the annulus
h, c	hot and cold wall
α	the direction of streaming step
$\overline{\alpha}$	the opposite direction of α

of the cylinders at the top region of the annulus. For the moderate-gap annuli, $1.24 < R < 1.71$, an oscillatory convection exists above a critical Ra, and it is characterized by a three-dimensional spiral motion in the upper portion of the annulus. Finally, for the narrow annuli, $R < 1.24$, two-dimensional multicellular convections appear as a result of instability of the crescent-shaped convection. This experiment was repeated and the classification was confirmed by Dyko et al. [10].

Many numerical simulations have also been carried out to study the natural convection instabilities and bifurcations in horizontal concentric annuli [11–15]. Yoo [11,12], in his outstanding work, reported the occurrences of dual solutions for Ra larger than a critical value. He observed dual steady solutions at $Ra > Ra_c - 3800$ for the wide gap annuli ($R = 2$) by vorticity-stream function method. Similar results were provided later by Chung et al. [13], and Desrayaud et al. [14]. Mizushima et al. [15] were the first to attempt a rigorous determination of the behavior of the system near the bifurcation point, confirming that the dual solutions were due to an imperfect transcritical bifurcation. In addition, the bifurcations phenomena in natural convection were theoretical analyzed by many researchers [16–20]. Busse [16] discussed the non-linear properties such as the dependence of the heat transport on Rayleigh and Prandtl numbers and the stability properties of thermal convection. Petrone et al. [17,18] performed a stability analysis of numerical steady-state solutions, and provided a detail of the bifurcation diagram near the imperfect bifurcation for $R = 1.2$, 1.4 and 2 at $Pr = 0.7$. Desrayaud et al. [19] proposed an analytical model for the stationary behavior of binary mixtures and pure fluids in a horizontal annular cavity. Angeli et al. [20] investigated the nonlinear dynamics of a confined buoyant flow.

As mentioned above, much work has been done for the bifurcation phenomena of natural convection in a horizontal concentric annulus. However, few studies concerned the eccentric annulus. In fact, Angeli et al. [21], in a critical review of buoyancy-induced flow transitions in horizontal annuli, pointed out that bifurcation phenomena in an eccentric annulus has not been studied. He predicted that the eccentricity parameter may be decisive for the competition between thermal and hydrodynamic instabilities. Therefore, in the present work, we investigate the bifurcation phenomena in eccentric annulus using the LBM and provide bifurcation diagrams for different eccentricities.

The lattice Boltzmann method (LBM) is a powerful numerical technique based on kinetic theory for simulating fluid flows and modeling the physical process in fluids. Owing to its mesoscopic origin, in comparison with conventional fluid dynamics solvers, it offers such advantages as simple calculation procedure, simple and efficient implementation for parallel computation, and easy and robust handling of complex geometries. In 1998, Chen and Doolen [22] first applied the LBM to the fluid flow problems. Then, it was developed to solve a wide range of heat transfer problems [23]. Quite recently, natural convection problems have also been investigated by the LBM [24]. Since LBM is inherently transient, it is an excellent approach for the investigation of thermal and hydrodynamic instabilities and bifurcations which are included in unsteady or periodic process.

Therefore, the present study is motivated by the need of considering bifurcation and flow structure maps in eccentric annulus in order to complete the research published for two-dimensional natural convection in a horizontal annulus. The remainder of this paper is organized as follows. In the next section, the lattice Boltzmann equations for natural convection is briefly outlined. Separate particle distribution functions in the LBM are used to calculate the density field, the velocity field and the thermal field. The curved boundary treatment is also detailed presented. In Section 3, the bifurcation phenomena for different Ra, Pr and R in concentric annulus are analyzed, and the results are compared with other published works to validate the LBM code. In Section 4, the main results of natural convection in eccentric annulus are obtained, and the pitchfork bifurcation diagrams are provided for different eccentricities. Finally, the conclusions are drawn in Section 5 of this paper.

2. Mathematic formulation

2.1. Physical model of eccentric annulus

As shown in Fig. 1, an eccentric annulus with an inner radius R_i and an outer radius R_o is considered. The eccentricity ε is defined as