International Journal of Thermal Sciences 89 (2015) 357–361

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Predicting radiation emitted from planar and fractal surfaces

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ARTICLE INFO

Article history:
Received 4 June 2014
Received in revised form
14 November 2014
Accepted 19 November 2014
Available online

Keywords:
View factor
Radiation emission
Planar surface
Fractal surface

ABSTRACT

Analytical solutions for the radiation view factor between either a planar surface or a plane-based fractal surface and an arbitrarily-positioned and arbitrarily-oriented receptive element were obtained. Deterministic fractal surfaces, whose planar cross sections are the Koch curve or a Cantor set, were considered. For surfaces facing each other the view factor exhibits a monotonic behavior as a function of the distance separating them. Otherwise, a maximum appears. Experiments using planar emitting surfaces were conducted which confirm this trend. It was also found that the non-monotonic behavior is strengthened for Cantor and Koch fractal surfaces. Size effects were discussed.

1. Introduction

In applications where radiation is the dominant mechanism of heat transfer (e.g. wildland and industrial fires, engineering applications, or illumination problems), the determination of view factors between surfaces is crucial. These include a wide variety of fire safety issues (e.g. evaluation of firefighter safety zones, assessment of fuel-break efficiency or flame radiation exposure at the wildland–urban interface). For surfaces facing each other analytical and numerical studies showed that radiation decreases monotonically with the distance between them [1–6]. However, these studies considered only canonical surfaces, generally planes or cylinders. In most applications the surface shape may be irregular or even fractal. For example in Ref. [7], Caldarelli et al. revealed the fractal nature of Mediterranean fire scars. Using the box counting method on satellite images [8], they found a fractal dimension of the fire perimeter of about 1.3. In the case of fractal surfaces, some parts of the emitting surface are screened by other parts, and thus shadow effects lead to a reduction in the radiation received by the target. A question arises: does this monotonic behavior still apply when the receptive element is arbitrarily positioned in space? The aim of this paper is to identify the conditions under which non-monotonic behavior occurs. Analytical view factor solutions are determined for planar and plane-based fractal emitting surfaces whatever the position of the receptive element in 3D space. In the present study we focus on deterministic fractal surfaces whose planar cross sections are either a Cantor set [9] or the Koch curve [10]. The former, with a fractal dimension of the cross section of 0.64 (less than 1), is representative of discontinuous surfaces, whereas the latter, with a fractal dimension of 1.26 (close to that obtained by Caldarelli et al. [7]), may mimic continuous rough surfaces with possible shadowing effects.

2. Analytical solutions of the view factor between a single panel and a receptive element

The knowledge of the view factor F from a finite surface S_2 to an infinitesimal surface element dS_1 allows to relate the heat flux (in W/m^2) leaving S_2 directly toward and intercepted by dS_1 (denoted q''_1) to the total heat flux (in W/m^2) leaving S_2 into all directions (denoted q''_2).

$$q''_1 = F \times q''_2 \tag{1}$$

The view factor (VF) is thus defined as.

$$F = \int_{S_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dS_2 \tag{2}$$

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http://dx.doi.org/10.1016/j.ijthermalsci.2014.11.024
1290-0729/

where $\cos\theta_1 = \vec{n}_1 \cdot \vec{r}/r$ and $\cos\theta_2 = \vec{n}_2 \cdot \vec{r}/r$. The notations used are given in Fig. 1a where the orthonormal coordinate system $(O\vec{x}\vec{y}\vec{z})$ is attached to the receptive surface element dS_1. In the present study we consider situations where the receptor is either parallel $(\vec{n}_1 = \vec{y})$ or perpendicular to the emission surface $(\vec{n}_1 = \vec{z}$ or $\vec{n}_1 = \vec{x})$, with $\vec{n}_2 = -\vec{y}$. After integrating over the emitting surface S_2, we obtain

- For the parallel case: $\vec{n}_1 = \vec{y}$

$$F^{\parallel} = \frac{1}{2\pi}\left\{\frac{x_{\min}+b}{C_x^b}\left[\tan^{-1}\left(\frac{z_{\min}+a}{C_x^b}\right) - \tan^{-1}\left(\frac{z_{\min}}{C_x^b}\right)\right] - \frac{x_{\min}}{C_x}\left[\tan^{-1}\left(\frac{z_{\min}+a}{C_x}\right) - \tan^{-1}\left(\frac{z_{\min}}{C_x}\right)\right] + \frac{z_{\min}+a}{C_z^a}\left[\tan^{-1}\left(\frac{x_{\min}+b}{C_z^a}\right) - \tan^{-1}\left(\frac{x_{\min}}{C_z^a}\right)\right] - \frac{z_{\min}}{C_z}\left[\tan^{-1}\left(\frac{x_{\min}+b}{C_z}\right) - \tan^{-1}\left(\frac{x_{\min}}{C_z}\right)\right]\right\} \quad (3)$$

- For the perpendicular case with $\vec{n}_1 = \vec{z}$

$$F_z^{\perp} = \frac{c}{2\pi}\left\{\frac{1}{C_z}\left[\arctan^{+}\left(\frac{x_{\min}+b}{C_z}\right) - \arctan^{+}\left(\frac{x_{\min}}{C_z}\right)\right] - \frac{1}{C_z^a}\left[\arctan^{+}\left(\frac{x_{\min}+b}{C_z^a}\right) - \text{atan}^{+}\left(\frac{x_{\min}}{C_z^a}\right)\right]\right\} \quad (4)$$

- For the perpendicular case with $\vec{n}_1 = \vec{x}$

$$F_x^{\perp} = \frac{c}{2\pi}\left\{\frac{1}{C_x}\left[\arctan^{+}\left(\frac{z_{\min}+a}{C_x}\right) - \arctan^{+}\left(\frac{z_{\min}}{C_x}\right)\right] - \frac{1}{C_x^b}\left[\arctan^{+}\left(\frac{z_{\min}+a}{C_x^b}\right) - \arctan^{+}\left(\frac{z_{\min}}{C_x^b}\right)\right]\right\} \quad (5)$$

where $C_z = \sqrt{c^2 + z_{\min}^2}$, $C_z^a = \sqrt{c^2 + (z_{\min}+a)^2}$, $C_x = \sqrt{c^2 + x_{\min}^2}$, and $C_x^b = \sqrt{c^2 + (x_{\min}+b)^2}$. The function $\arctan^{+}$ corresponds to the positive part of the inverse tangent function (i.e, $\arctan^{+}(x) = \tan^{-1}(x)$ if $x > 0$, and 0 otherwise). It is easy to show that the VFs given by Eqs. (3)–(5) only depend on the dimensionless ratios a/c, b/c, $x_{\min}/c$, and $z_{\min}/c$, which is particularly useful for scaling purposes.

The solution for an arbitrarily-oriented and arbitrarily-positioned receptive element is a combination of perpendicular and parallel VFs.

$$F = \max[(\vec{n}_1 \cdot \vec{x}); 0]F_x^{\perp} + \max[(\vec{n}_1 \cdot \vec{y}); 0]F^{\parallel} + \max[(\vec{n}_1 \cdot \vec{z}); 0]F_z^{\perp} \quad (6)$$

Eq. (6) generalizes the analytical expressions found in the literature for receptive elements located in front of the emitting surface [3–6]. For $x_{\min} = z_{\min} = 0$, Eq. (3) or Eq. (4) reduce to the view factor solution derived by Hamilton [3], and Eq. (1) to that given by Hollands [4]. For $x_{\min} = b/2$ and $z_{\min} = a/2$, Eq. (2) is similar to McGuire's expression [5].

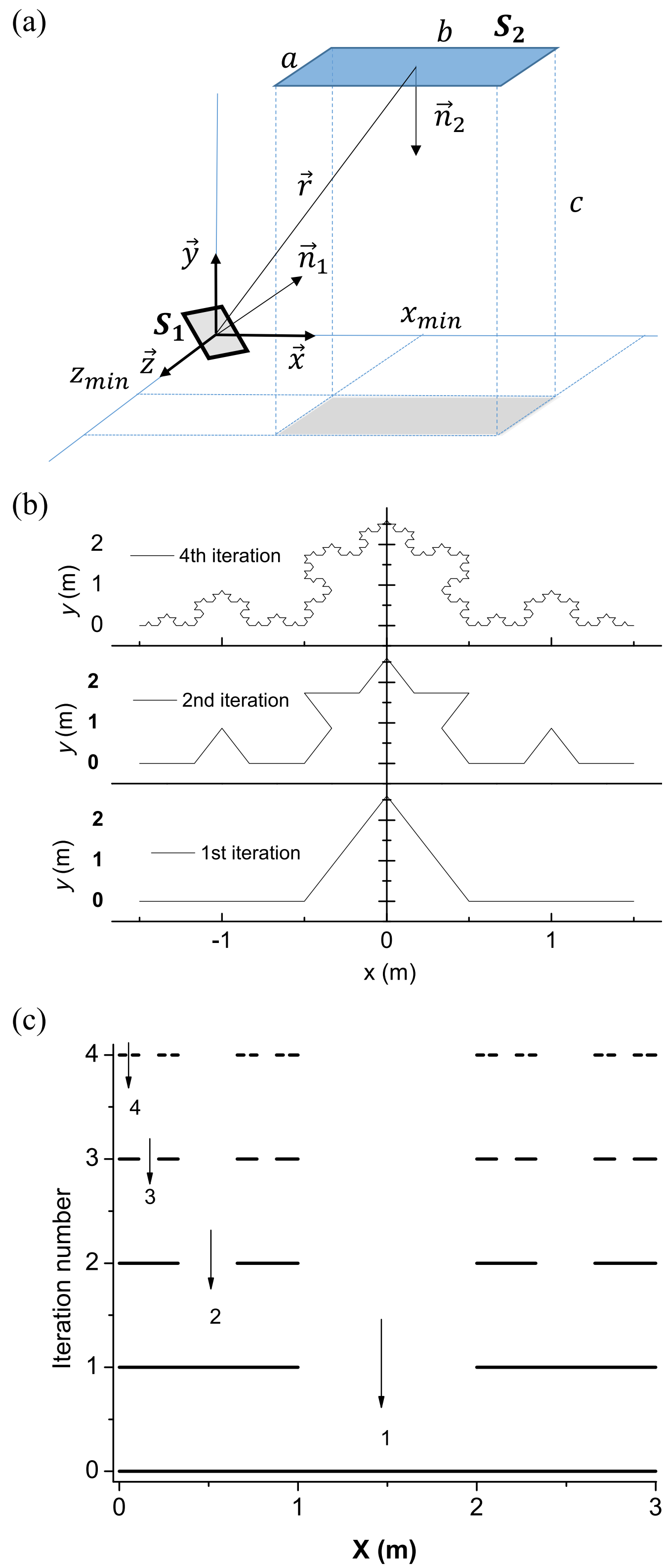

Fig. 1. (a) Geometry and notations used for view factor calculations; (b) and (c) cross sections of Koch-like and Cantor-like surfaces at successive iterations. The length of the radiant system is $b = 3$ m.