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Nano fluid flow in tapering stenosed arteries with permeable walls

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A B S T R A C T

This problem deals with the theoretical study of nano fluid flow through composite stenosed arteries with permeable walls. The highly non linear momentum equations of nano fluid model are simplified by considering the mild stenosis case. The solutions for concentration and temperature are found by using homotopy perturbation method (HPM) while exact solution for velocity profile is calculated. Moreover the expressions for flow impedance, pressure rise and stream function are computed and discussed through graphs for different physical quantities of interest.

1. Introduction

Blood is the bodily fluid which delivers nutrients and oxygen to the cells and transport waste products away. Blood circulates around the body through vessels and arteries by pumping action of heart. In cardiac related problems, the effected arteries get hardened as a result of accumulation of fatty substances inside the lumen or because of formation of plaques. These accumulation of substances in arteries known as stenosis. These stenosis narrow the artery because of which blood has to pass with relatively high pressure.

The vessel or arterial walls may be elastic, movable or permeable. Therefore to understand the mechanics of circulation of the blood, it would be prerequisite to have a clear idea of basic mechanics of fluid. Some of the basic studies dealing different models of Newtonian, non-Newtonian fluid are given in Refs. [1–6] For last couple of decades, researcher have become interested to model the blood flow in stenosed arteries experimentally and theoretically. A number of theoretical studies related to the blood flow through stenosed arteries have been carried out recently in which most of the studies focused in presence of mild or single stenosis [7,8]. The mathematical modeling of pulsatile flow of Herschel Bulkely fluid in stenosed arteries has been examined by Sankar and Lee [9]. They used regular perturbation technique and found analytic solutions. The Newtonian behavior of blood flow is also discussed in their article. Study on three layered oscillatory blood flow through stenosed arteries is discussed by Tripathy [10]. Mekheimer and Elkot [11] have discussed the mathematical modeling of unsteady flow of sisko fluid through an anisotropically tapered elastic arteries with time variant overlapping stenosis. Many researchers have highlighted about arteriosclerotic development which indicate that the studies are mainly concerned with the single symmetric and non symmetric stenosis. Akbar and Nadeem [12,13] analyzed the blood flow of different fluids in stenosed arteries. Recently, Mishra et al. [14] have studied the blood flow through a composite stenosis in an artery with permeable wall. For further analysis see Refs. [15,16].

The aim of present paper is to see the effects of permeable walls along with slip on the nano fluid flow through tapered arteries with stenosis. The non-dimensional governing equations in the case of mild stenosis and corresponding boundary conditions are prescribed and then solved by using homotopy perturbation method (HPM). The physical features of the major parameters have been discussed through the graphs. Trapping phenomenon have also been discussed at the end.

2. Formulation of the problem

Consider an incompressible nano fluid of viscosity μ and density ρ flowing through a tube of finite length L with overlapping stenosis. Let (r, θ, z) be the coordinates of a material point in the cylindrical polar coordinate system where z-axis is taken along the axis of artery while r, θ are along the radial and circumferential

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direction respectively. Moreover $r = 0$ is taken as the axis of symmetry of the tube. Heat and mass phenomenon are taken into account by giving temperature T_1 and concentration C_1 to the wall of the tube. At the centre of the tube, we consider symmetric conditions at temperature and concentration. The geometry of the arterial wall of the overlapping stenosis for different taper angles is written mathematicaly [12]

$$R(z) = d(z)\left[1 - \psi\left(L_0^{n-1}(z-d_0) - (z-d_0)^n\right)\right], \quad d_0 < z \le d_0 + L_0 \tag{1}$$

$$R(z) = d(z), \quad \text{otherwise} \tag{2}$$

with

$$\psi = \frac{\delta n^{\frac{n}{n-1}}}{R_0 L_0^n (n-1)}$$

$$d(z) = R_0 + \xi z.$$

Here δ denotes the maximum height of the stenosis located at

$$z = d_0 + \frac{L_0}{n^{\frac{n-1}{n}}}.$$

R_0 is the radius of the non tapered artery in the non stenotic region, $d(z)$ is the radius of the tapered arterial segment in the stenotic region, ξ is the tapering parameter, L_0 is the length of the stenosis, n (≥ 2) is a parameter determining the shape of the constriction profile and referred to as the shape parameter, for which symmetric stenosis is found for $n = 2$ and d_0 indicates its location as shown in Fig. 1.

The equations for unsteady flow of an incompressible nano fluid in the presence of body force are given

$$\nabla \cdot \mathbf{V} = 0 \tag{3}$$

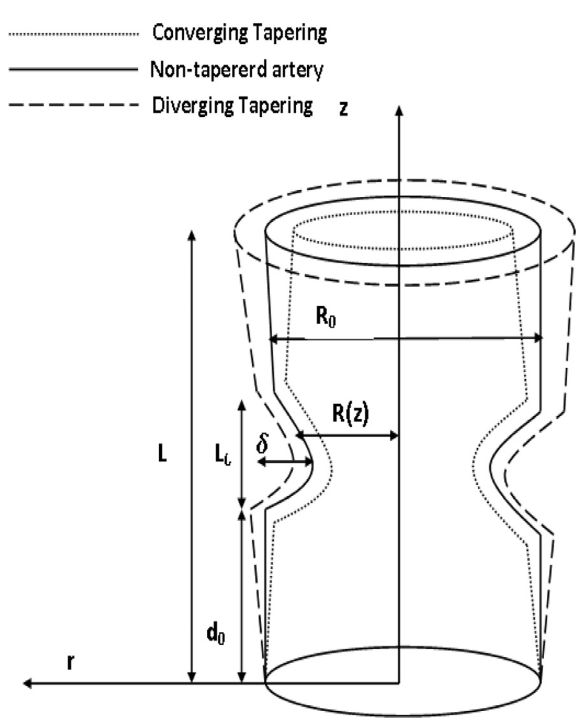

Fig. 1. Geometry of the stenosed tapered artery for different taper angle.

$$\rho_f \frac{dV}{dt} = -\nabla P + \mu \nabla^2 V + \mathbf{f}, \tag{4}$$

$$(\rho c)_f \frac{dT}{dt} = k\nabla^2 T + (\rho c)_p [D_B \nabla C \cdot \nabla T + (D_T/T_0)\nabla T \cdot \nabla T], \tag{5}$$

$$\frac{dC}{dt} = D_B \nabla^2 C + (D_T/T_0)\nabla^2 T, \tag{6}$$

c is the volumetric volume expansion coefficient, $\mathbf{V}$ is the velocity vector, $\mathbf{f}$ is the body force, d/dt represents the material time derivative, P is the pressure C is the nanoparticle phenomena, the ambient values of T and C as r tends to R are denoted by T_1 and C_1, D_B is the Brownian diffusion coefficient and D_T is the thermospheric diffusion coefficient. In component form Eqs. (3) to (6) can be written as

$$\frac{1}{r}\frac{\partial (rv)}{\partial r} + \frac{\partial u}{\partial z} = 0, \tag{7}$$

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right), \tag{8}$$

$$\rho\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g\alpha(T - T_1) + \rho g\alpha(C - C_1), \tag{9}$$

$$\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = \alpha\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right) + \tau\left[D_B\left(\frac{\partial C}{\partial r}\frac{\partial T}{\partial r} + \frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right) + \frac{D_T}{T_0}\left(\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial r}\right)^2\right)\right] \tag{10}$$

$$\left(v\frac{\partial C}{\partial r} + u\frac{\partial C}{\partial z}\right) = D_B\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right) + \frac{D_T}{T_0}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) \tag{11}$$

where $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nano particle material and heat capacity of the fluid.

The boundary conditions for temperature and concentration are

$$\frac{\partial T}{\partial r} = 0, \quad \frac{\partial C}{\partial r} = 0 \quad \text{at } r = 0, \tag{12}$$

$$T = T_1, \quad C = C_1 \quad \text{at } r = R(z). \tag{13}$$

We introduce the following non-dimensional variables

$$\bar{r} = \frac{r}{R_0}, \quad \bar{z} = \frac{z}{L_0}, \quad \bar{v} = \frac{L_0}{\delta U}v, \quad u = \frac{u}{U}, \quad \bar{R} = \frac{R}{R_0}, \quad \bar{p} = \frac{R_0^2}{UL_0\mu}p$$
$$T = T_1 + (T_0 - T_1)\theta, \quad C = C_1 + (C_0 - C_1)\sigma, \quad G_r = \frac{\rho g\gamma R_0^2 T_1}{\mu U},$$
$$B_r = \frac{\rho g\gamma R_0^2 T_1}{\mu U}, \quad N_t = \frac{(\rho c)_p D_T T_0}{(\rho c)_f \gamma}, \quad N_b = \frac{(\rho c)_p D_T C_0}{(\rho c)_f \gamma}, \quad R_e = \frac{\rho U R_0}{\mu} \tag{14}$$

where U is the velocity averaged over the section of the tube with radius R_0, N_t, N_b, G_r and B_r are the thermophoresis parameter, the Browian motion parameter, local temperature Grashof number, local