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Scaling and direct stability analyses of natural convection induced by absorption of solar radiation in a parallelepiped cavity

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ABSTRACT

The present study considers the early stage of the flow development in a water-filled, three-dimensional parallelepiped cavity subject to solar radiation. The thermal structure consists of an upper stable stratification due to internal heating, provided by the direct absorption of radiation, and a bottom potentially unstable boundary layer due to the absorption and re-emission of the residual radiation by the bottom boundary. The growth of the competing thermal layers and the stability properties of the bottom thermal boundary layer are investigated by means of a scaling analysis and a direct stability analysis based on three-dimensional numerical simulation. The scaling relations are benchmarked against the numerical simulation, and the effects of the normalised water depth, the Rayleigh number and the box aspect ratio on the thermal instability are studied.

1. Introduction

Buoyancy driven flows are the basis of many engineering, environmental and meteorological applications. Therefore, a large body of literature exists for classical problems, including the Rayleigh–Bénard and Rayleigh–Bénard–Poiseuille flows (e.g. Refs. [1–3]), boundary layer flows in enclosures or over a semi-infinite plate (e.g. Refs. [4–8]), and plume flows (e.g. Refs. [9–13]), with an imposed boundary temperature or heating flux. The flow stability, which is often an important part of the studies in this field, has been investigated based on a linear stability analysis, and a substantial review for boundary layer flows and plumes is provided in Ref. [14]. More recently, a direct stability analysis, in which the full Navier–Stokes equations, instead of linearised equations, are solved numerically with the addition of small artificial perturbations, has been successfully employed to obtain the stability properties of a natural convection boundary layer in a differentially heated cavity [6] and those of a solar radiation induced natural convection boundary layer in a triangular cavity [15]. The latter has direct relevance to the present study and also to studies on the stability of convection flows driven by a combination of internal heating and boundary effects (e.g. Refs. [16–19]).

The natural convection flows driven by internal heating have important implications in a wide range of applications, e.g. convection in Earth's mantle, astrophysical plasmas [19], and convection in atmospheres, oceans and lakes [17]. The significance of natural convection flows in the control of water quality and biological activity in littoral water bodies has also been highlighted in Refs. [20] and [21]. In the present study, the stability of natural convection flow in a parallelepiped cavity, with water as the working fluid, induced by the absorption of solar radiation is investigated. In this problem, the internal heating is provided by the absorption of the incident radiation by a water body, hence the radiation intensity is attenuated with depth. The attenuation of the radiation intensity in water bodies follows Beer's law and depends on the wavelength of the incident radiation, water colour and transparency. However, the present study uses a bulk value of the attenuation coefficient, η, as in previous studies (e.g. Refs. [22–24]) and therefore the vertical (y) distribution of the radiation intensity (I) is given as [25]:

$$I = I_0 e^{\eta y} \quad (y \leq 0), \tag{1}$$

with $I = I_0$ at $y = 0$ (the bulk radiation intensity at the water surface). A typical attenuation coefficient η for inland waters is in the range $O(1)$ ~ $O(10)$ m^{-1} [26]. The radiation thermal forcing is instantaneously increased to and maintained at a fixed rate. Clearly, the absorption of radiation leads to a temperature profile which exponentially decays from the surface. In the absence of other effects, this leads to a stable stratification. It is assumed that any residual radiation reaching the bottom is absorbed and re-emitted as a boundary heat flux, leading to the formation of a potentially

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unstable thermal boundary layer [22]. These two effects suggest the existence of two distinct thermal layers; a surface layer with a stable stratification and a bottom boundary layer with a potentially unstable stratification, which is a possible source for a Rayleigh–Bénard type instability. The relative significance of these competing thermal layers depends on the comparison between the water depth (h) and penetration depth of the incident radiation (η^{-1}). In shallow waters, with relatively small values of $h\eta$ ($\lesssim 1$), the bottom thermal boundary layer dominates while in deep waters, with relatively large values of $h\eta$ (>1), the radiation is mostly absorbed in the surface layer with only a small fraction of the radiation reaching the bottom. The vertical temperature profile in the absence of any flow, which is a function of $h\eta$ and time t, was obtained by solving the one-dimensional conduction equation with the source term associated with the internal heating and flux boundary conditions in Ref. [22].

The flow in a radiatively heated triangular cavity has been investigated previously. Lei and Patterson [27] reported a flow visualisation experiment of radiation induced natural convection using the shadowgraph technique and observed the instability in the form of rising plumes from the bottom thermal boundary layer. The later direct numerical simulations [28] provided qualitative agreement with the experimental observations, and the three-dimensional nature of the instability was demonstrated. The stability properties of the boundary layer were investigated in Ref. [15] as already mentioned. A scaling analysis was also carried out [23], which was later improved [24] by including position dependence into the previous scaling. The numerical verification confirmed the validity of the scaling analysis.

The previous numerical and experimental studies are all limited to the case with $h\eta < 1$. The scaling analysis [24] identifies the possible flow regimes depending on $h\eta$ and the bottom slope, however, the numerical verification was only provided for the case with $h\eta = 0.12$. The direct stability analysis in Ref. [15] was also focused on the shallow water case with $h\eta = 0.37$. From the studies on the stability of internally heated convection flows [16,17], it is expected that the stability properties are strongly affected by the vertical temperature profile and therefore by the value of $h\eta$. A linear stability analysis of [22] shows the dependence of the critical condition for the formation of rolls on $h\eta$, assuming a laterally unbounded fluid layer. However, it is well known [1,29] that lateral confinement greatly affects the stability properties of convection flows, and in real situations, including laboratory scale experiments, the flows are always confined. Therefore, further investigation of the effect of the parameter $h\eta$ on the flow development and stability in a confined three-dimensional cavity would be useful, which motivated the present study.

The significant variation associated with the difference in the geometry from the previous studies for a triangular cavity discussed above is that with a triangular cavity a convection flow up the slope develops due to the horizontal temperature gradient [30], which limits the growth of the bottom boundary layer, whereas with a parallelepiped cavity with no bottom slope considered in the present study, there is no horizontal convection flow in the early stage, as for the case of the classical Rayleigh–Bénard convection. Therefore the properties of the instability of the thermal structure can be investigated in isolation. The effect of $h\eta$ on the growth of the competing thermal layers is investigated by means of a scaling analysis, whereas its effect on stability properties is investigated by means of a direct stability analysis based on three-dimensional numerical simulation. Further, the effect of the horizontal aspect ratio is also investigated, which has been studied for the classical Rayleigh–Bénard problem [29]. The present study focuses on the early stage of the flow development. The later stage of the flow development, characterised by intermittent convection [31], is beyond the scope of this paper and will be the focus of a future study.

2. Model formulation

The schematic of the model is provided in Fig. 1. The lateral dimensions are defined by L_x and L_z and the vertical dimension of the box, i.e. the water depth, is denoted by h. With the Boussinesq approximation and constant water properties assumed at $T_0 = 293$ K, the equations governing the radiation induced natural convection flow are given as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\nabla^2 u, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \nu\nabla^2 v + g\beta T, \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} + \nu\nabla^2 w, \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \kappa\nabla^2 T + S(y), \tag{5}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{6}$$

where u, v and w are the velocity components in the x, y and z directions as indicated in Fig. 1; T is the temperature perturbation with respect to the spatially averaged temperature as explained below; and p is the pressure. The density, kinematic viscosity, thermal diffusivity and thermal expansion coefficient at T_0 are ρ_0, ν, κ and β, respectively. Following Beer's law (Equation (1)), the internal heating source term in Equation (5) due to the direct absorption of solar radiation is defined as [15]:

$$S(y) = \frac{I_0}{\rho_0 C_p}\eta e^{\eta y} - \frac{I_0}{\rho_0 C_p h} \quad (y \le 0), \tag{7}$$

where I_0 is the radiation intensity at the water surface ($y = 0$), C_p is the specific heat of water at T_0, and η is the bulk attenuation

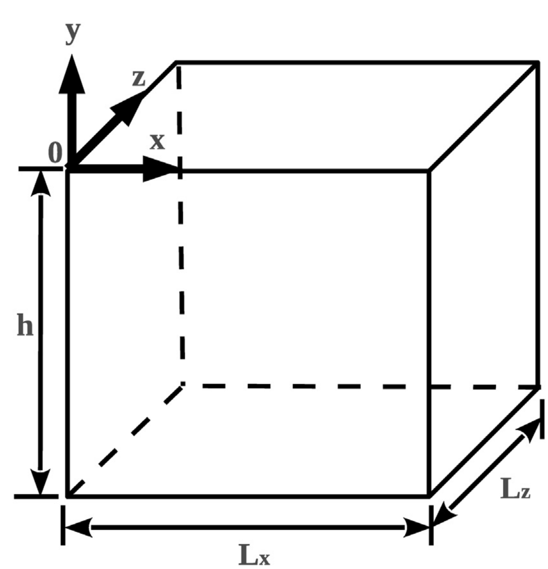

Fig. 1. Geometry of the flow domain.