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Analytical solution of discontinuous heat extraction for sustainability and recovery aspects of borehole heat exchangers

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Existing analytical solutions for thermal analysis of ground source heat pump (GSHP) systems evaluate temperature change in the carrier-fluid and the surrounding ground in the production period of a single borehole heat exchanger (BHE) only if a continuous heat load is assigned. In the present study, we modified the Green's function, which is the solution of heat conduction/advection/dispersion equation in porous media, for discontinuous heat extraction by analytically convoluting rectangular function or pulses in time domain both for single and multi-BHEs field. The adapted analytical models for discontinuous heat extraction are verified with numerical finite element code. The comparison results agree well with numerical results both for conduction and advection dominated heat transfer systems, and analytical solutions provide significantly shorter runtime compared to numerical simulations (approx. 1500 times shorter). Furthermore, we investigated the sustainability and recovery aspects of GSHP systems by using proposed analytical models under different hydro-geological conditions. According to the engineering guideline VDI 4640, a linear relationship between thermal conductivity of the ground and the sustainable heat extraction rate is demonstrated for multi-BHEs.

1. Introduction

GSHPs are known as renewable and sustainable energy systems for decades [1–7]. In order to keep the performance of these systems suitable in decades, a proper installation should be planned for BHE, and to re-operate them after an operation period, the ground needs time to recover from temperature drop. This is particularly the case for multi-BHEs that may affect significantly the ground temperature on a relatively large area.

Rybach and Eugster [3] carried out 2D numerical studies on the sustainability and renewability aspects of a single BHE in a long-term performance in Switzerland. The first 11 years operation of a BHE, measured data are set as the load profile of heat extraction, and an additional load profile of 19 years is extrapolated according to the meteorological data. They showed the temperature decrease in the ground at different distances apart from the BHE at the depth of 50 m during the production period of 30 years and subsequent 30 years as the recovery phase of the ground. According to their results, during the production period the temperature change of the ground was not less than ~7 K at the distance of 0.3 m, and when the operation is stopped, the ground is quickly recovered in several years (e.g. Temperature change of the ground = ~1 K), and after the recovery phase of 30 years the temperature drop was nearly 0.1 K regarding to undisturbed ground temperature. Signorelli [4] investigated the sustainability of BHEs with 3D numerical studies for both a single and multi-BHEs (6 BHEs, array spacing between 3 and 15 m). The temperature change at 50 m depth and 0.1 m distance from the BHE was less than 0.1 K after a 70 years recovery period for multi-BHE field with the array spacing of 7.5 m, whereas after 24 years for a single BHE. Lazzari et al. [8] studied on the long-term performance of single and multi-BHEs field with periodic (sinusoidal) heat load only in a conduction dominated heat transfer system under the consideration of two different ground thermal conductivity. Particularly, they pointed out that even with a large space distance between BHEs (14 m), the full compensation of the ground temperature is needed in case of multi-BHEs field (i.e. in winter heating, in summer cooling). Furthermore, such a similar study under groundwater flow is carried out by Zanchini et al. [9]. They showed that with increasing of Péclet number (proportional to the groundwater flow), the performance of BHEs field becomes more sustainable.

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Nomenclature

a	thermal diffusivity (m^2/s)
c	specific heat capacity (J/kg/K)
H	borehole length (m)
Q_P	energy extraction or injection (J)
Q_L	heat input per meter depth (J/m)
q_L	heat input rate per unit length of borehole (W/m)
t	time (s)
T	temperature (K)
v_T	thermal transport velocity (m/s)
u_x	darcy's velocity (m/s)
x, y, z	space coordinates (m)

Greek symbols

α_l	longitudinal thermal dispersion coefficient
α_t	transversal thermal dispersion coefficient
λ_m	bulk thermal conductivity of porous medium (W/m/K)
λ_x	effective thermal conductivity in the longitudinal direction (W/m/K)
$\lambda_y = \lambda_z$	effective thermal conductivity in the transverse direction (W/m/K)
ρ	density (kg/m^3)

Subscripts

m	medium
w	water

Those previous studies already showed that GSHP systems provide reliable performance and can be used as renewable energy source. However, this is validated only for one-type of geology. In a long-term operation of a BHE, the sustainable heat extraction and the ground temperature recover after the operation depend on hydro-geological and thermo-physical characteristics of the ground.

In order to investigate the scenarios mentioned above with recovery periods of BHEs after an operation, current appropriate methods are to evaluate temperature change in the ground with numerical studies. However, the numerical simulations of GSHP systems are 3D problem and if the discontinuous operation is taken into account, it requires powerful computers due to large computational effort. On the other hand, most of the analytical solutions described in literature consider a constant continuous heat extraction/injection in time merely for a single BHE [10–16], or discontinuous heat extraction under different conditions [10,17,18]. Eskilson [10] described a simple analytical solution for discontinuous heat extraction neglecting three-dimensional effect, but his equation provides only sinusoidal oscillations of temperature signal depending on the assigned distance apart from a single source. Claesson and Eskilson [17] presented 1D analytical solution of a finite line heat source to analyze dimensionless temperature change of subsurface for single pulse, and pulsated heat extraction rate. The temperature solution is obtained for any piecewise constant heat extraction value by superposition using Duhamel's theorem [19]. In case of multiple heat sources, Claesson and Probert [20] derived an analytical solution for temperature field of heat releasing canisters containing nuclear waste. The local field temperature is solved with a finite line heat source and infinite grid of point sources. The solutions for different heat sources are superimposed. Recently, the long-term performance of BHEs field has been investigated with an analytical solution that is extended for sinusoidally varying heat load in an infinite medium neglecting the groundwater flow [21].

The main objective of this study is to obtain an analytical solution to evaluate temperature change in the ground both for single and multi-BHEs that considers discontinuous heat extraction, thermal conduction, advection and dispersion. We start from the Green's function which is the solution of heat conduction/advection/dispersion equation in porous media and apply an analytical convolution of that function with a rectangular function or pulses, which have different period lengths and pulse heights. The evolution of the mean fluid temperature of the carrying fluid to maintain a constant heat extraction rate is evaluated along the time. Temperature evaluation in the surrounding ground is also deduced. The developed equation is verified with the finite element software COMSOL Multiphysics. Furthermore, the sustainability of single/multi-BHEs under different hydro-geological conditions is investigated with analytical solution to model 30 years production period, and also from an environmental point of view, the recovery period after those 30 years of operation is studied.

2. Analytical solution for discontinuous heat extraction

2.1. Single BHE

In geothermal literature, the existing finite and cylindrical analytical solutions with a constant heat load may provide satisfactory estimation of ground thermal parameters to design GSHP systems [22–24]. In a real case, the GSHP systems can be operated with various periods in a given time for different heat extraction/injection rates, instead of a continuous operation as assumed by most of other previously presented analytical methods. Some authors evaluated the temperature change for thermal response test operation in the vicinity of a single BHE or BHE field with an analytical solution by using multiple load aggregation algorithms [25–30]. However, some of those approaches may not be appropriate in all cases to evaluate the accurate temperature change in the ground due to neglecting axial effect, considering only single BHE or not taking into account groundwater flow. In particular when Darcy's velocity in porous media is considered, the thermal dispersion coefficients must be taken into account, because thermal dispersion has a large impact on the distribution of the temperature plume around BHE, for Darcy's velocity larger than 10^{-8} m/s [31].

The governing equation of the heat conduction/advection/dispersion in porous media is given as follows:

$$\rho_m c_m \frac{\partial T}{\partial t} = \left(\lambda_x \frac{\partial^2 T}{\partial x^2} + \lambda_y \frac{\partial^2 T}{\partial y^2} + \lambda_z \frac{\partial^2 T}{\partial z^2} \right) - u_x \rho_w c_w \frac{\partial T}{\partial x} + s \tag{1}$$

in which u_x is the Darcy's velocity on the x-direction, s is a volumetric heat source, and $\rho_m c_m$ is the volumetric heat capacity of the medium, which can be calculated as the weighted arithmetic mean of the solids $\rho_s c_s$ and volumetric heat capacity of water $\rho_w c_w$ [32]:

$$\rho_m c_m = (1-n)\rho_s c_s + n \rho_w c_w \tag{2}$$

where n is the porosity.

The components of effective longitudinal and transverse thermal conductivities are defined on the directions of x, y and z as follows [33,34]:

$$\lambda_x = \lambda_m + \alpha_l \rho_w c_w u_x \tag{3}$$

$$\lambda_y = \lambda_z = \lambda_m + \alpha_t \rho_w c_w u_x \tag{4}$$

where λ_m is the bulk thermal conductivity of porous medium in the absence of groundwater flow, α_l and α_t are the longitudinal and