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A new parameter for the dynamic analysis of building walls using the harmonic method

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A B S T R A C T

The authors propose a methodology to schematize correctly the capacitive effects in the transmission of heat in the multilayered walls of buildings.

An analytical study is presented related to a steady periodic regime allowing consideration of three external loads acting singularly or simultaneously: air temperature, apparent sky temperature and incident solar irradiation.

Such a study is applied in the case of four traditional types of wall (A – brick wall, B – hollow wall, C – polarized brick wall, D – prefabricated wall).

The expression of the oscillating heat flux, which penetrates the internal environment, and the conductive heat flux which penetrates the wall in contact with the external air, was obtained by means of the electrical analogy and the resolution of the equivalent circuit. It is demonstrated that the nondimensional periodic global transmittance, the ratio between the heat flux which is transferred to the indoor environment and the external heat flux, with the plant turned on, is the most suitable nondimensional parameter for the dynamic analysis of the walls. This parameter allows for the evaluation of all the typical dynamic quantities for the complete description of the thermal behavior of the walls.

1. Introduction

In the air-conditioning of buildings, the spread of cooling systems has determined a significant increase in annual energy requirement. In order to contain the energy consumption, as indicated by the recent regulations regarding energy efficiency [1,2], it is necessary to improve the thermal performance of the building shell and use more efficient plant systems. In many cases the evaluation of heat fluxes, through the external components of the building shell, is carried out with simplified procedures when one is interested in the determination of energy uses [3], while plants dimensioning requires the determination of the peak powers, which are obtained using calculation codes which can simulate the effective behavior of the building air conditioning system in dynamic conditions.

Often, during the preliminary phase of thermal design and the performance evaluation and diagnosis of buildings, the use of simplified models results as being convenient [4,5]. In Refs. [6,7] it is highlighted how, in the formulation of simplified models, it is important to correctly schematize the thermal capacitive effects and identify the parameters necessary to describe accurately the phenomena of thermal exchange and storage.

A further issue, which can arise during the planning phase, consists in the evaluation of the influence of the stratigraphical composition of the walls on the dynamic properties of the building shell [8,9]. In Refs. [10–13] the time lag and the decrement factor of a wall are determined by varying the thermophysical properties of the materials, the thickness and the position of the insulation layer, considering the sol-air temperature as the external load with a sinusoidal trend equal to 24 h. A similar approach was adopted in Ref. [14] in which an analytical study on the influence of the color, or rather of the optical properties, of the external surfaces of a building component on the transfer of heat in a steady periodic regime is presented.

A sophisticated model that takes into account the heat transfer and moisture transfer in walls whose layers are made with nonhomogeneous materials is shown in Ref. [15].

In Refs. [16,17] traditional methods for the resolution of the conduction equation (numerical methods, harmonic methods, response factor methods and methods based on conduction transfer functions (CTF)) are compared, and the advantages and disadvantages of each method are shown.

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In particular, the harmonic method presents the advantage of providing analytical expressions of the parameters which identify the dynamic behavior of the building components. Such a setting is found in recent Standards such as EN ISO 13792:2012 [18] and EN ISO 13786:2010 [19]. The latter uses harmonic analysis in a steady periodic regime for the dynamic characterization of building components. The boundary conditions on the two faces which delimit the wall are temperature or heat flux that vary sinusoidally.

The shell walls are generally subjected to variable loads over time, prevalently due to the external air temperature, to the incident solar radiation and to the infrared radiation from the sky. From a mathematical standpoint, it is possible to approximate the variation law of such loads over time as the sum of infinite contributions (harmonics) whose variations are sinusoidal over different periods. The first is called fundamental harmonic and has period $P = 24$ h; the successive ones have $P_k = P/k$ with k integer.

The resolution of the general equation of conduction is obtained in the entire domain and, by means of the heat transfer matrix, allows the calculation of the complex amplitudes of the temperature and the heat flux on the internal side, starting from the complex amplitudes of the loads which act on the external surface.

The characteristics used are the periodic thermal admittances and the properties of dynamic heat transfer; specifically, the periodic thermal admittance correlates the oscillating heat flux with the oscillation of the temperature on the same face of the component, while the dynamic heat transfer properties correlate the oscillations of a quantity on a face of the component with the corresponding one on another face.

Among the dynamic heat exchange properties, the Standard [19] considers the periodic thermal transmittance, a complex number which provides the decrement factor of the amplitude of the oscillation and the time lag to which the load is subjected when it crosses the component. The periodic thermal admittances and transmittance are used to determine the areal heat capacity which quantifies the storage properties. It is possible to define an admittance and a thermal capacity on each side for each component.

In this work, the dynamic characterization of the walls, in steady periodic regime, is widened through the consideration of three distinct loads: the external air temperature, the apparent sky temperature and the incident solar radiation. Moreover, a dynamic analysis of the building component is developed considering the joint action of the three periodic loads, with the formulation of a matrix expression, obtained by means of the electrical analogy and the overlapping of the causes and effects, which permits the calculation of the heat flux which appears within the indoor environment when the three loads act externally.

The procedure was specially created and used in order to identify further characteristic dynamic parameters, with the aim of describing the heat transfer phenomena in a more complete manner.

Verification of the results obtained with the analytical procedure was carried out by means of a numerical model of the wall using the finite difference method.

The external loads trend was obtained starting from the experimental data of the external variables, air temperature, solar irradiation and sky temperature, recorded in clear sky conditions.

2. Methodology

The experimental values of each external load are expressed in an analytical form through a discrete Fourier series [20] with interruption of the tenth harmonic, which approximates the data by a mean steady value and a sum of sinusoids of different frequencies, amplitude and argument according to the equation:

$$y(t) = \bar{y} + \sum_{k=1}^{10} \tilde{y}_k = \bar{y} + \sum_{k=1}^{10} |\tilde{y}_k| \sin(k\omega t + \psi_k) \tag{1}$$

in which $\bar{y}$ represents the mean value, $\tilde{y}_k$ the amplitude, $k\omega$ the angular frequency and ψ_k the argument of the k-th harmonic. Fig. 1 reports the experimental data of the loads and the trends, obtained by setting the angular frequency ω equal to 0.262 rad/s, corresponding to a period of 24 h, and the steady value equal to the mean in the entire period.

In Tables 1–3 the mean value and the characteristic parameters of the first ten harmonics are reported.

Starting from the entering signals and the transfer matrix of the wall, the symbolic or phasors method is applied which, for each generic harmonic $\tilde{y}_k$ supposes the entering signal as an imaginary part of the more general signal [21]:

$$\hat{y}_k = |\hat{y}_k|[\cos(k\omega t + \psi_k) + j\mathrm{sen}(k\omega t + \psi_k)] = |\hat{y}_k| e^{j(k\omega t + \psi_k)} \tag{2}$$

Once the exiting signal has been obtained in its complex form, it is necessary to multiply it by the complex operator $e^{jk\omega t}$ and select solely the imaginary part, which forms the solution for the generic harmonic of angular frequency $k\omega$. By summing the responses obtained for the various harmonics, and adding the term relative to the null angular frequency component (steady conditions), the response to the entering signal, of which the Fourier finite series was initially broken down, is obtained.

Therefore, an entering fixed sinusoid is first transformed into a complex form and then multiplied by the transfer matrix of the same period, in order to obtain the corresponding exiting sinusoid in a complex form. In order to pass from a complex domain to a time domain, it is necessary to consider the imaginary part of this result.

3. Analysis of the dynamic response of a wall to single loads

The Standard EN ISO 13786:2010 [19] exemplifies the dynamic characterization of building components considering the external air temperature of a period equal to 24 h as a load. The convective-radiative heat transfer coefficient for internal and external surface heat exchanges are used for these evaluations. In the present work, a more accurate investigation is developed and the characterization of the building component is obtained considering the three loads and effective boundary conditions on the external surface singularly, whereas on the internal surface the thermal exchange was modeled by means of the surface heat coefficient.

For each load, the contribution of a single harmonic is evaluated considering the relation between the complex amplitudes of the temperature and of the heat flux inside and those corresponding which act on the outside [22]. The expressions obtained are:

a) external air temperature ("ea" load)

$$\begin{bmatrix} \hat{T}_{\mathrm{ia,ea}} \\ \hat{\varphi}_{i,\mathrm{ea}} \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{1}{h_i} \\ 0 & 1 \end{bmatrix} [Z] \begin{bmatrix} 1 & -\dfrac{1}{h_{e,c}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{T}_{\mathrm{ea}} \\ \hat{\varphi}_{e,c} \end{bmatrix} = \begin{bmatrix} S_{11,\mathrm{ea}} & S_{12,\mathrm{ea}} \\ S_{21,\mathrm{ea}} & S_{22,\mathrm{ea}} \end{bmatrix} \begin{bmatrix} \hat{T}_{\mathrm{ea}} \\ \hat{\varphi}_{e,c} \end{bmatrix} = S_{\mathrm{ea}} \begin{bmatrix} \hat{T}_{\mathrm{ea}} \\ \hat{\varphi}_{e,c} \end{bmatrix} \tag{3}$$

b) apparent sky temperature ("*sky*" load)