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Simultaneously regular inversion of unsteady heating boundary conditions based on dynamic matrix control

Yanhao Li [a], Guangjun Wang [a, b, *], Hong Chen [a, b]

[a] School of Power Engineering, Chongqing University, Chongqing 400044, PR China
[b] Key Laboratory of Low-grade Energy Utilization Technologies and Systems, Chongqing University, Ministry of Education, Chongqing 400044, PR China

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ABSTRACT

Based on the dynamic matrix control (DMC) idea, a method is established to simultaneously estimate boundary heat flux for unsteady heat conduction system in this paper. The measured temperature information at two measured points in the internal system is utilized to simultaneously determine transient heat flux $q_1(t)$ and $q_2(t)$ at two boundaries in the system. The algorithm adopts step response function to describe dynamic relationship of the system, without prior supposing the surface heat flux in the next period, and then the boundary heat flux at the present moment is inversed simultaneously through rolling optimization. In order to give attention to both the stability of inversion results, the inversing heat flux $q_1(t)$ and $q_2(t)$ are regularized respectively using different regularization parameters α_1 and α_2. A method of same temperature discrepancy curve (STDC) is designed to estimate the optimal values $(\alpha_1)_{opt}$ and $(\alpha_2)_{opt}$ of regularization parameters according to discrepancy principle. Numerical experiments are divided into two parts. First of all, compared with the sequential function specification method (SFSM), the effects of the future time steps as well as the measured temperature error on simultaneously inversion results of the boundary heat flux are investigated. Results show that the DMC inversion method established by this paper can use smaller future time steps r to simultaneously estimate transient boundary heat flux of the system, and obviously improve the anti-interference of measured noise. Second, the validity of the inversion results obtained by the proposed method is discussed respectively by cases with different sizes and changing rules of boundary heat fluxes as well as different measured locations. And by comparing with the traditional centralized regularization (CR), the necessity of respectively regularization for each inversing heat flux is confirmed by using different regularization parameters.

1. Introduction

It is the direct heat conduction problems to obtain temperature field by solving heat conduction differential equation with the object's thermal physical parameters, initial conditions and boundary conditions known. If some boundary conditions and initial conditions, etc. are unknown in the direct heat conduction problem, the temperature changing laws of some known points are utilized to inverse the unknown definite conditions, which is the inverse heat conduction problems (IHCP). IHCP has been widely applied in the fields of power engineering, aerospace, biological heat transfer, etc. [1–5].

In recent decades, people have developed and improved many solution strategies of IHCP from different angles [6–8]. A numerical solution of IHCP is established according to using the Tikhonov regularization method (TRM) combined with the conjugate gradient method (CGM) considered by Lu et al. [9]. As an effective numerical solution, the sequential function specification method (SFSM) proposed by Beck et al. [10–12] is especially widely used in the unsteady IHCP. Wang et al. [13–15] proposed a decentralized fuzzy inference (DFI) method for the inverse problems of steady heating boundary conditions, significantly improving the anti ill-posed characteristic of inverse process. Zhang et al. [16] applied the CGM to inverse spatially and temporally varying boundary temperature in two-dimensional heat conduction system.

In the above all algorithms, the SFSM and TRM have received more applications [6,17–19]. It is necessary for SFSM to firstly assume specific function forms of the inversing parameters over a period of future time, and the measured temperature information

* Corresponding author. School of Power Engineering, Chongqing University, Chongqing 400044, PR China. Tel.: +86 23 65103512.
E-mail address: wangguangjun@cqu.edu.cn (G. Wang).

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in the future period of time is used to inverse boundary conditions at the current time, which is one effective way to solve the unsteady IHCP because of high computational efficiency. For SFSM, it is very sensitive to the future time steps r, the selection of r has a direct influence on the anti-interference of inversion results for measured error [1]. With the increasing of measured temperature error, a larger future time steps r is required in order to guarantee the stability of inversion results. In addition, it is also an obvious disadvantage for SFSM to necessarily assume the specific functions of the inversing parameters over a period of future time. For TRM, the traditional centralized regularization (CR) is introduced into the objective function to improve the stability of inversion results [1,3]. However, considering for estimating multiple heating boundary conditions simultaneously (for example, different sizes of boundary heat fluxes), TRM has adopted the same regularization parameter to estimate all heating boundary conditions, which could cause instability of inversion results of partial heating boundary conditions. Namely, all heating boundary conditions can't be effectively identified simultaneously, which needs to be improved for TRM.

As a kind of important predictive control algorithm, Dynamic matrix control (DMC) has been widely applied in the field of modern control [20]. The choosing of control amounts at the current moment is determined by rolling optimization according to predictive information of the system's outputs over a period of future time. DMC has the characteristics of strong adaptability and good robustness, etc.

Based on DMC idea, an inversion method of simultaneously regularization is established for two-dimensional unsteady heating boundary conditions in this paper. The system's step response is adopted as the prediction model, and the measured temperature information is used to simultaneously identify heat flux of different boundaries, and then an optimization method about regularization parameters based on a method of same temperature discrepancy curve (STDC) is established, the inversing heat fluxes are regularized respectively using different regularization parameters. Compared with SFSM and traditional CR, the rationality of the inversion method established in this paper is proved.

2. Two-dimensional unsteady heat conduction model

The two-dimensional unsteady heat conduction system is used as the object of study as shown in Fig. 1. It is assumed that the evenly distributed and transient boundary heat fluxes are applied to the surface S1 and S2 respectively. Namely, the two surface heat fluxes q_1 and q_2 are two functions with only respect to the time variable t. The governing equation, the initial and boundary conditions of temperature $T(x,y,t)$ are as follows respectively:

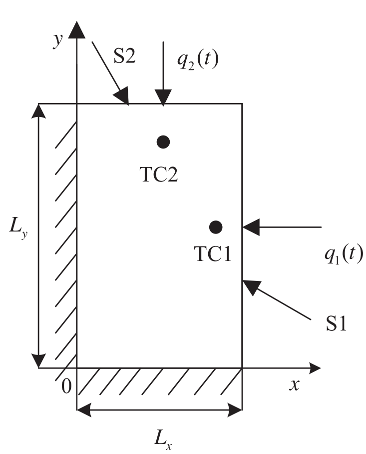

Fig. 1. Two-dimensional unsteady heat conduction system.

$$\rho c \frac{\partial T(x,y,t)}{\partial t} = \lambda \left[\frac{\partial^2 T(x,y,t)}{\partial x^2} + \frac{\partial^2 T(x,y,t)}{\partial y^2} \right], \quad 0 \le x \le L_x, \ 0 \le y \le L_y, \ t > 0, \tag{1a}$$

$$T(x,y,t) = T_0(x,y), \quad 0 \le x \le L_x, \ 0 \le y \le L_y, \ t = 0, \tag{1b}$$

$$-\lambda \frac{\partial T(x,y,t)}{\partial y} = 0, \quad 0 \le x \le L_x, \ y = 0, \ t > 0, \tag{1c}$$

$$-\lambda \frac{\partial T(x,y,t)}{\partial x} = 0, \quad x = 0, \ 0 \le y \le L_y, \ t > 0, \tag{1d}$$

$$-\lambda \frac{\partial T(x,y,t)}{\partial x} = q_1(t), \quad x = L_x, \ 0 \le y \le L_y, \ t > 0, \tag{1e}$$

$$-\lambda \frac{\partial T(x,y,t)}{\partial y} = q_2(t), \quad 0 \le x \le L_x, \ y = L_y, \ t > 0, \tag{1f}$$

where ρ is the density, c and λ are the heat capacity and thermal conductivity coefficient respectively, $q_1(t)$ and $q_2(t)$ are respectively boundary heat flux on the surface S1 and S2, $T_0(x,y)$ is the initial temperature distribution.

In the condition of all known in above, Eq. (1) is solved by the finite volume method and alternating direction implicit algorithm to determine transient temperature distribution $T_{i,j,k}$ in the system. Here, $T_{i,j,k} = T(x_i,y_j,t_k)$, which shows temperature of the node (x_i,y_j) at the moment t_k.

3. Dynamic matrix control (DMC) algorithm for solving inverse problem

If other conditions are known in Eq. (1), but the boundary heat flux q_1^k and q_2^k are unknown, the boundary heat flux q_1^k and q_2^k are identified according to the measured temperature values of the measured points TC1 and TC2 at moment $k, k+1, \cdots, k+r-1$, which has constituted the corresponding IHCP. Here r stands for the predetermined future time steps, which are the numbers of time discrete points over a period of future from the current moment kth. q_1^k and q_2^k are the estimated values of $q_1(t)$ and $q_2(t)$ respectively at the current moment$k$.

The aforementioned IHCP is solved by utilizing DMC algorithm in this work.

3.1. Establishing prediction model

The step response of the object is adopted as prediction model to predict the temperature $T_m^{k+j} (j = 0, 1, \cdots r-1; \quad m = 1, 2)$ of r moment at the measured point m according to increments of boundary heat flux $\Delta q_i^k, \Delta q_i^{k+1}, \cdots, \Delta q_i^{k+r-1} \ (i = 1, 2)$ in the future moment in the system.

According to the superposition principle, the input increments Δq_i^{k+j} with groups of r are input to the system from the kth moment, the temperature T_m^{k+j} at the measured point m is the sum of the output $\hat{T}_m^{k+j}$ at the $(k+j)$th time without any control increments and the outputs when the r control increments is applied individually to the system, namely: