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On the thermal instability induced by viscous dissipation

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ABSTRACT

The effect of viscous dissipation may be the sole cause of a thermoconvective instability either in a fluid clear of solid material or in a fluid-saturated porous medium. Several recent investigations have contributed to illustrate this result under different flow and thermal conditions. The elementary physical nature of the dissipation induced instability is just the same as that of the Rayleigh–Bénard instability, namely the onset of a secondary buoyant flow taking place when the basic temperature gradient becomes sufficiently intense. The essential difference is that the dissipation instability is not induced by an external thermal forcing due to the temperature boundary conditions, as it happens for the Rayleigh–Bénard instability. On the other hand, the cause of the dissipation instability is the basic flow rate itself, acting thermally as a heat generation mechanism. Thus, the governing parameter determining the transition from stability to instability is not the Rayleigh number, as in the classical Rayleigh–Bénard problem, but the product between the Gebhart number and the square Péclet number.

1. Introduction

The thermoconvective instability, either in a fluid or in a fluid-saturated porous medium, is usually associated with a vertical temperature gradient prescribed through the boundary conditions. The prototypical setup is the well-known Rayleigh–Bénard system: a fluid at rest confined between two horizontal plane boundaries having uniform temperatures, and such that the lower boundary is hotter than the upper boundary. The Rayleigh number, or the Darcy–Rayleigh number in the case of porous media, serves to establish the onset conditions for the instability through its critical value: when this critical value is exceeded, convective cells appear in the fluid initially at rest.

Although it is not so uncommon to consider viscous dissipation in fluid flows as a necessarily negligible effect, this is not always the case. A recurrent argument is that one cannot warm up a cup of cold coffee by stirring it with a spoon. On the contrary it is possible, provided that one uses a suitable cup and a suitable spoon. The first evidence of this possibility dates back to circa 1850, when James Prescott Joule published the results of his very famous paddlewheel experiment about the *mechanical equivalent of heat*. Actually, what Joule showed was that it is possible to rise the temperature of water in a thermally insulated tank (the cup) with paddles (the spoon) whose rotation is caused by a falling weight. Despite its usual interpretation relevant to the first principle of thermodynamics, one may well recognise that Joule's experiment is the first evidence of the intensity of viscous dissipation, under suitable flow conditions.

Recent studies [1–22] have shown that the thermoconvective instability may be produced just by the effect of viscous dissipation, even without a boundary thermal forcing, as in the Rayleigh–Bénard system. In other words, the basic vertical temperature gradient is built up solely by the internal viscous heating, while the thermal boundary conditions act just as to allow the downward orientation of the gradient. A pair of boundary conditions that produces this effect is adiabatic lower boundary and isothermal upper boundary. An important point is that, unlike in the Rayleigh–Bénard system, a linear instability induced by viscous dissipation can exist only if the basic state of the fluid is one with a nonvanishing velocity field. In fact, in the classical Rayleigh–Bénard system, the viscous dissipation would just produce a higher-order nonlinear effect, negligible in a linear stability analysis. It must be mentioned that the idea of an instability induced by viscous dissipation in shear flows is not novel, as Joseph [23] described a similar effect more than forty years ago. However, the instability studied by Joseph is not a thermoconvective one, as it is not induced by the thermal buoyancy. On the other hand, it is caused by the temperature-dependence of the fluid viscosity.

The main aim of this contribution is to illustrate the main results obtained on the dissipation-induced thermoconvective instability. Starting from the modelling of viscous dissipation in the framework of the Oberbeck–Boussinesq approximation, the role played by this

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Nomenclature

a	wave number
(a_x, a_z)	wave vector
c	specific heat
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
c_f	form-drag coefficient
$\mathcal{D}$	strain tensor, $(\mathcal{D}_{ij})$
$\boldsymbol{e}_y$	unit vector along the y-axis
f, h	perturbation amplitude functions
$\boldsymbol{F}_d$	drag force
g	modulus of $\boldsymbol{g}$
$\boldsymbol{g}$	gravitational acceleration
Ge	Gebhart number
k	thermal conductivity
$\tilde{k}$	effective thermal conductivity
K	permeability
L	layer thickness
p	pressure
P	dynamic pressure, $p - \rho_0 \boldsymbol{g} \cdot \boldsymbol{x}$
Pe	Péclet number
q_g	power generated per unit volume
R	Darcy–Rayleigh number
Ra	Rayleigh number
t	time
T	temperature
T_c, T_h	boundary temperatures
T_0	reference temperature
$\boldsymbol{u}$	velocity, (u, v, w), (u_i)
$\boldsymbol{U}$	velocity perturbation, (U, V, W)
u_0	reference velocity
x, y, z	Cartesian coordinates
$\boldsymbol{x}$	position vector, (x_i)

Greek symbols

α	thermal diffusivity
$\tilde{\alpha}$	effective thermal diffusivity
β	thermal expansion coefficient
γ	ratio a_x/a
ε	perturbation parameter
η	dimensionless parameter, Eq. (48)
θ	temperature perturbation
Λ	product $GePe^2$
μ	dynamic viscosity
$\tilde{\mu}$	effective dynamic viscosity
ν	kinematic viscosity
ρ	density
ρ_0	reference density
σ	heat capacity ratio
φ	porosity
Φ	power dissipated per unit volume
χ	inclination angle
ψ	scalar field, Eq. (40)
ω	angular frequency

Subscripts

cr	critical value

effect in determining the linear stability bound will be described, both with reference to clear fluids and to saturated porous media. Several cases will be surveyed, providing numerical results and suggesting possible comparisons with studies of the thermo-convective instability in heat generating fluids [24–27].

Another objective of this paper is to report novel results regarding the onset of dissipation-induced instability in a fluid saturated porous layer with horizontal throughflow and an upper open boundary. This analysis is a development of the study previously carried out by Barletta, Celli, and Rees [2] with reference to a similar problem where the upper boundary of the porous layer is assumed to be impermeable.

2. The Oberbeck–Boussinesq approximation and the local energy balance

2.1. A clear fluid

Let us consider a clear fluid, *viz.* a fluid clear of solid material, one whose momentum balance is given by the Navier–Stokes equation. The conceptual scheme for describing buoyant flows is the Oberbeck–Boussinesq approximation. The nature of the Oberbeck–Boussinesq approximation stems from the assumption that the fluid properties are considered as constants with the only exception of the density, ρ, whose change is taken into account only in the gravitational body force term of the momentum balance. The linear equation of state is in fact assumed,

$$\rho = \rho_0[1 - \beta(T - T_0)], \tag{1}$$

where the reference density ρ_0 corresponds to a properly defined reference temperature T_0, and β is the thermal expansion coefficient at constant pressure.

Equation (1) implies that the density is evaluated at constant pressure and that the temperature changes are very small. Thus, the mass and momentum balance equations are given by

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho_0} \nabla P - (T - T_0)\beta \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u}, \tag{3}$$

where $\boldsymbol{u} = (u_i)$ is the velocity field, t is time, and $\boldsymbol{x} = (x_i)$ is the position vector. Moreover, ν is the kinematic viscosity, $\boldsymbol{g} = (g_i)$ is the gravitational acceleration, T is the temperature field, while P is the dynamic pressure, namely the difference between the pressure field p and the static pressure field, $\rho_0 \boldsymbol{g} \cdot \boldsymbol{x}$. Equations (2) and (3) must be completed with the energy balance in order to achieve the closure of the problem. In the literature, there is a manifold answer to the question of the proper formulation of the local energy balance in the framework of the Oberbeck–Boussinesq approximation. In fact, one may have Chandrasekhar's [28] and White's [29] formulation

$$\rho_0 c_v \left(\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T \right) = k \nabla^2 T + q_g + 2\mu \mathcal{D} : \mathcal{D}, \tag{4}$$

where $\mu = \rho_0 \nu$ is the dynamic viscosity, k is the thermal conductivity, c_v is the specific heat at constant volume. The symbol $\mathcal{D}$ denotes the strain tensor, having components

$$\mathcal{D}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{5}$$

while $\mathcal{D} : \mathcal{D}$ stands for the double dot product $\mathcal{D}_{ij} \mathcal{D}_{ij}$, where the summation over repeated indices is assumed.