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Transient conjugated heat transfer in microchannels: Integral transforms with single domain formulation

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ABSTRACT

The transient behaviour of conjugated heat transfer in laminar microchannel flow is investigated, taking into account the axial diffusion effects, which are often of relevance in microchannels, and including pre-heating or pre-cooling of the region upstream of the heat exchange section. The solution methodology is based on the Generalized Integral Transform Technique (GITT), as applied to a single domain formulation proposed for modelling the heat transfer phenomena at both the fluid stream and the channel wall regions. By making use of coefficients represented as space dependent functions with abrupt transitions occurring at the fluid–wall interfaces, the mathematical model carries the information concerning the transition of the two domains, unifying the model into a single domain formulation with variable coefficients. The proposed approach is illustrated for microchannels with polymeric walls of different thicknesses. The accuracy of approximate internal wall temperature estimates deduced from measurements of the external wall temperatures, accounting only for the thermal resistance across the wall thickness, is also analyzed.

1. Introduction

Transient forced convection in channels has been a subject of intense research activity since the earlier works of S. Kakaç and collaborators [1–3], in the context of fundamental work aimed at the development of analytical solution techniques and the experimental validation of proposed models and methodologies, for both the fully transient and periodic states, in either laminar or turbulent regimen [4–16]. This research front was later on further pursued in the direction of extending the previously developed hybrid tools to handle both transient flow and transient convection problems in microchannels within the slip flow regime [17–22].

Most of these works on transient forced convection were based on extension or application of the ideas in the Generalized Integral Transform Technique (GITT), a well-established hybrid numerical-analytical solution methodology for convection-diffusion problems [23–28]. Also, most of the works cited above do not account for wall conjugation effects or at most consider only the influence of the wall thermal capacitance on the transient behavior of the fluid temperature evolution along the channel [7–9]. A few works have analyzed the effects of heat conduction along the wall, in addition to thermal capacitance, in the transient behavior of internal forced convection with wall participation [29–35].

Although transients might be very rapid during convective heat transfer within microchannels, due to the short length and time scales involved, conjugation effects might play a major role in the transient behavior of thermal microsystems, considerably increasing the duration of the thermal responses to different timewise disturbances. Conjugated heat transfer is in fact a typical example of an effect that may have significant importance in micro-scale convective heat transfer but is often neglected in macro-scale situations. Nunes et al. [36] presented a theoretical-experimental study of steady state conjugated heat transfer in microchannels, with the theoretical model taking into account heat conduction along the microchannel walls length, extending the work developed in Ref. [37]. This work is also based on the Generalized Integral Transform Technique, modeled through a mixed lumped-differential thermal formulation which proposes lumping over the wall transversal direction only and accounting for the longitudinal heat conduction. The approach in Ref. [36] yields simulation

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Nomenclature

Bi	Biot number
D_h	hydraulic diameter
g_i	source term in the transformed problem, Eq. (16a)
h_e	heat transfer coefficient at the channel external wall
K	dimensionless thermal conductivity
k	thermal conductivity
L	thickness of the channel wall
L_e	distance from the channel centerline to the external face of the channel wall
L_f	channel height
L_w	channel width
M	truncation order of the auxiliary problem
N_ζ, N_ξ, N_Ω, N_ϕ	Normalization integrals corresponding to the eigenfunctions ζ, ξ, $\Omega$and ϕ
N	truncation order of the temperature field expansions
Pe	Péclet number
T	temperature field
t	time variable
U	dimensionless fully developed velocity profile
u	velocity profile
W	dimensionless heat capacity
w	heat capacity
Y	dimensionless transversal coordinate
y	transversal coordinate
Z	dimensionless longitudinal coordinate
z	longitudinal coordinate

Greek letters

α	thermal diffusivity, Eq. (2)
β,α,λ,υ,μ	eigenvalues corresponding to ξ,ζ,Ω, ϕ, and ψ respectively
δ	transition function
ζ	eigenfunction of the insulated region eigenvalue problem
η	parameter that controls the transition spatial behavior
θ	dimensionless temperature field
ν	kinematic viscosity
ξ	eigenfunction of the heat exchange section eigenvalue problem
τ	dimensionless time
ϕ	eigenfunction of the auxiliary problem corresponding to insulated region eigenvalue problem
ψ	eigenfunction
Ω	eigenfunction of the auxiliary problem corresponding to the heat exchange section eigenvalue problem

Subscripts & superscripts

ad	quantity corresponding to the insulated upstream region
i,*m*,*n*	order of eigenquantities
$-$	integral transform
$\sim$	normalized eigenfunction
$_f$	fluid
$_s$	solid
in	channel inlet
∞	external environment

results in much better agreement with the available experimental data obtained in the same work, reconfirming the importance of wall conjugation in micro-channels applications.

More recently, the reformulation of conjugated conduction-convection problems has been proposed as a single region model that fully accounts for the local heat transfer at both the fluid flow and the channel wall regions [38–40]. This novel approach allows for a straightforward hybrid analytical-numerical analysis of more involved conjugated heat transfer problem formulations, again based on the GITT in either total or partial transformation schemes. By introducing coefficients represented as space variable functions with abrupt transitions occurring at the fluid–wall interfaces, the mathematical model carries the information concerning the two original domains of the problem.

In the present work, making use of this single domain formulation and the integral transform method, the transient behavior of conjugated heat transfer in laminar microchannel flow is investigated, taking into account the thermal capacitance and transversal and axial diffusion effects at both the fluid and the walls, and including the participation of the wall and fluid regions upstream of the heat exchange section. For illustration purposes, an existing experimental configuration is considered, consisting of a rectangular microchannel etched on a polyester resin substrate [41], with different wall thicknesses. We also verify the accuracy and merits of a simple thermal resistance model, applied across the wall thickness, in estimating the internal wall temperatures from available measurements of the external wall temperatures.

2. Problem formulation and solution methodology

Fig. 1 depicts a schematic representation of the problem under consideration. The channel wall is considered to participate in the heat transfer problem through both transversal and longitudinal heat conduction. We consider a laminar incompressible internal flow of a Newtonian fluid inside a rectangular channel with height and width given by L_f and L_w, respectively, being $L_w \gg L_f$ so that it can be represented by a flow between parallel plates, undergoing conjugated heat transfer between the fluid and bounding walls. The external face of the microchannel exchanges heat with the surrounding environment by means of convection with a known heat transfer coefficient, h_e. It is also considered that the microchannel heat exchange section, $0 \le z \le z_\infty$, may exchange heat with the transversally insulated upstream region, $-z_{\mathrm{ad},\infty} \le z \le 0$. The fluid flows with a fully developed velocity profile $u_f(y)$, and with uniform inlet temperature, T_{in}. The flow is assumed to be hydrodynamically

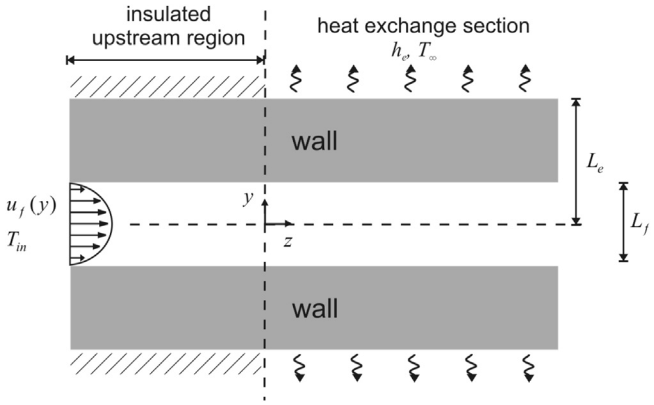

Fig. 1. Schematic representation of the transient conjugated heat transfer problem in a microchannel with upstream region participation.

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