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Steady and transient heat transfer analysis using a stable node-based smoothed finite element method

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ABSTRACT

In order to cure the instability of NS-FEM and further improve the accuracy, a stable node-based smoothed finite element method (SNS-FEM) is formulated for steady and transient heat transfer problems using linear triangular and tetrahedron element. In present method, both smoothed temperature gradient and variance of temperature gradient in smoothing domains are considered. The accuracy, computational efficiency and stability of SNS-FEM are examined through several numerical examples with different kinds of boundary conditions. It is found that present method is more accurate and efficient than traditional finite element method (FEM) and NS-FEM. Most importantly, compared with NS-FEM, present SNS-FEM can be very stable when dealing with transient heat transfer problems.

1. Introduction

The analysis of heat transfer problems is of great importance in practical engineering areas. As analytical solutions for these problems with complex geometry and boundary conditions are usually not available, numerical methods have been developed and extended for heat transfer problems in the past several decades.

The finite element method (FEM) has been used for heat transfer problems for a long time [1]. Donea and Giuliani [2] have utilized an FEM based iterative method for numerical solution of nonlinear heat transfer problems. Payre et al. [3] has developed an ‘upwind’ finite element method via numerical integration to improve previous upwind finite element schemes for diffusion-convection problems. The boundary element method (BEM) has been used by Skerget et al. [4] to solve transient heat transfer in reactor solids. The element free Galerkin (EFG) method has been applied to analyze heat transfer problems by Singh et al. [5–7]. Cheng et al. [8] applied the reproducing kernel particle method (RKPM) to two-dimensional unsteady heat conduction problems. MLPG method for two-dimensional steady-state heat conduction problems has been investigated by Wu et al. [9]. Feng et al. [10] has applied FS-PIM to analyze nonlinear heat conduction in multi-material bodies.

Among all these numerical methods mentioned above, finite element method is the most widely used numerical method with many commercial software packages available. However, the FEM always provide a poor accuracy when low-order linear element is used. Besides, FEM will suffer a significant accuracy loss when the element mesh is heavily distorted due to its strong reliance on element meshes. In the FEM framework, researchers have studied and developed many modified numerical algorithms to circumvent some of the defects and great progress has been made. Chen et al. [11] proposed the concept of strain smoothing technique, Liu et al. [12] have introduced this gradient smoothing technique into FEM framework. A class of smoothed FEM models have been developed on the base of the gradient smoothing technique, such as the smoothed finite element method (SFEM) (Liu et al. [13], Xue et al. [14]), the node-based smoothed finite element method (NS-FEM) (Liu et al. [15]), the edge-based smoothed finite element method (ES-FEM) (Liu et al. [16], Cui et al. [17,18], Feng et al. [19]) and the face-based smoothed finite element method (FS-FEM) (Feng et al. [20]). NS-FEM wins the favor recently for its prominent inherent properties: (1) can give an upper bound solution in energy norm for problems with homogeneous essential boundary conditions; (2) can use polygonal elements with an arbitrary number of sides; (3) well immune from volumetric locking; (4) insensitive to element distortion. Besides, researchers have studied the computation time

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and computation efficiency of NS-FEM in previous works and have found that the computational efficiency of NS-FEM is about three times lower than that of FEM-T3 in terms of displacement norm and 20 times higher in strain energy norm.

In the other front of development, Liu et al. [21] and Zhang et al. [22] have applied the nodal integration and strain smoothing technique into the point interpolation method (PIM), resulting the linearly conforming point interpolation method (LC-PIM or NS-PIM). The NS-PIM has been applied to steady heat transfer problems by Wu [23–25]. The NS-PIM using linear shape function is equivalent to the NS-FEM based on 3-node triangular element. The NS-PIM or NS-FEM has the same prominent properties in the analysis of steady heat transfer problems as in analysis of solid mechanics problems. However, the excessive node-based smoothing operation results “under-integration” of the weak form and makes the system stiffness “overly-soft”, resulting the temporal instability of the NS-FEM when solving transient heat transfer problems.

The temporal instability of nodal integration is firstly found in mesh-free method known as the nodal integration of the element free Galerkin method [26]. Beissel and Belytschko treated it by adding a squared-residual of the equilibrium equation to the potential energy functional as a stabilization term. Zhang et al. [27], Feng et al. [28] and Wang et al. [29] have further applied this technique to the NS-FEM solutions. A parameter α is introduced in this stabilization process to adjust the stiffness of system. And researchers have found it effective using linear elements. To strengthen the “overly-soft” stiffness of NS-FEM, other researchers have formulated the alpha finite element method (αFEM) (Liu et al. [30]) as well as the hybrid smoothed finite element method (HS-FEM) (Xu et al. [31], Li et al. [32]). They found that these methods can provide “ultra-accurate” numerical solutions with a proper parameter α. The parameter α plays an important role in all the methods mentioned above. Its value can greatly influence the numerical results. However, the value of α is affected by different factors, such as the size of mesh discretization and the nature of the problem. Until now, researchers have not found an optimal α value suitable for all problems.

In this work, a new stable node-based smoothed finite element method (SNS-FEM) is formulated for steady and transient heat transfer problems using linear triangular and tetrahedron element. The temperature gradient smoothing operation is firstly implemented within each smoothing domain as in NS-FEM, and then the temperature gradient is expanded at first order by Taylor equation. In each smoothing domain, both smoothed temperature gradient and variance of temperature gradient are considered. Numerical examples with different kinds of boundary conditions are presented to examine the stability, accuracy, as well as efficiency of present method through comparing the results with those obtained by the FEM and the original NS-FEM. Results show that present SNS-FEM can provide stable solutions for transient heat transfer problems, and performs better in accuracy and efficiency than traditional FEM and original NS-FEM.

2. Discretized system equations

2.1. Standard Galerkin weak form

To note that, 3D problem will be discussed in detail in the following, and in 2D equations, terms with respect to z or w will vanish, which can be easily obtained following the procedure.

Consider a three-dimensional heat transfer domain Ω bounded by Γ in which the governing equation is written as:

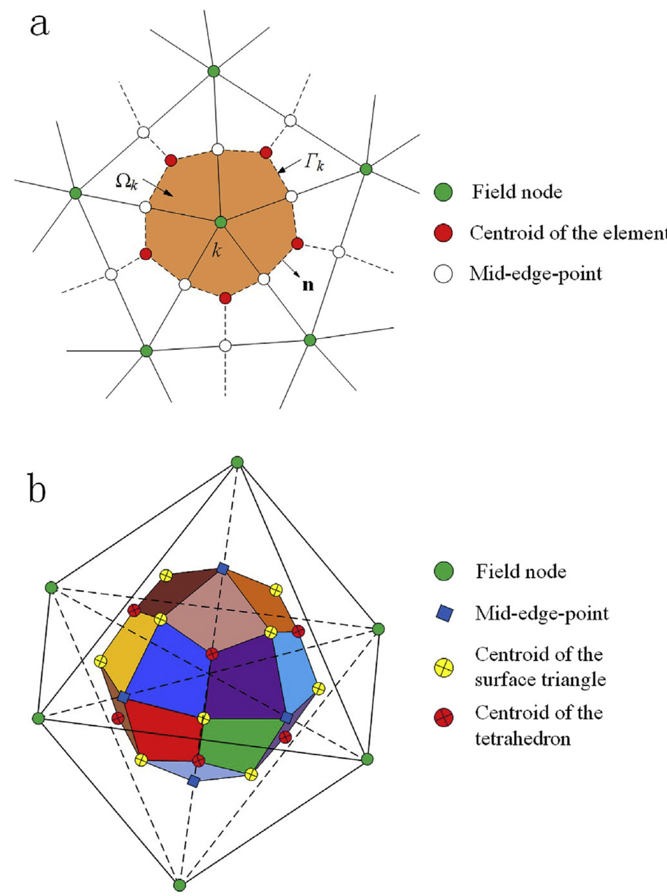

Fig. 1. The schematic of a node-based smoothing domain for node k. (a) 2D smoothing domain, and (b) 3D smoothing domain.

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) \quad \text{Problem domain studied}$$
$$+Q(x,y,z,t)=\rho c\frac{\partial T(x,y,z,t)}{\partial t} \tag{1}$$

The initial condition

$$T=T_0 \quad \text{Initial condition} \tag{2}$$

The boundary conditions

$$T=T_\Gamma \quad \text{Dirichlet boundary} \tag{3}$$

$$-k_x\frac{\partial T}{\partial x}n_x-k_y\frac{\partial T}{\partial y}n_y-k_z\frac{\partial T}{\partial z}n_z=q \quad \text{Neumann boundary} \tag{4}$$

$$-k_x\frac{\partial T}{\partial x}n_x-k_y\frac{\partial T}{\partial y}n_y-k_z\frac{\partial T}{\partial z}n_z=h(T-T_a) \quad \text{Robin boundary} \tag{5}$$

$$k_x\frac{\partial T}{\partial x}n_x+k_y\frac{\partial T}{\partial y}n_y+k_z\frac{\partial T}{\partial z}n_z=\mathbf{0} \quad \text{Adiabatic boundary} \tag{6}$$

Where k_x, k_y and k_z are the thermal conductivities, $Q(x,y,z,t)$ is the internal heat source, $T(x,y,,z,t)$ is the temperature of point (x,y,z) at time t, ρ is the density, c is the specific heat, n is the unit normal vector to the boundary, q is the prescribed heat flux, h is the