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Analytical solutions of non-Fourier bio-heat conductions for skin subjected to pulsed laser heating

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ABSTRACT

The analytical solutions of bio-heat transfer for skin tissue with general boundary conditions in the Pennes, Cattaneo-Vernotte (C-V) and Dual-phase-lag (DPL) models are presented. The heat transfer of skin subjected to the pulse laser heating and fluid cooling are studied. In the conventional literature, the inner boundary of skin will be assumed to be in the constant temperature condition or the isolated condition and their numerical difference is discussed here. The effects of the phase lags with respect to heat flux and temperature gradient on the thermal wave speed are investigated. The comparison of the presented results and the approximated speed formula, $\sqrt{k/\tau_q \rho c}$, in the C-V model is made. The skin damage index in several damage models by pulsed laser heating is discussed. Later, the discussion of effects of several boundary conditions on temperature variation and thermal damage in the Pennes, CV and DPL models is introduced. It is found that the procedure including pulsed heating and cooling creates the temperature fluctuation of skin. It is suitable for the cold and heat therapy for pain relief and other functions. The conditions of pulsed heating skin and simultaneous cooling for the thermal therapy are investigated. The analytical method can be widely applied for solving the general problem of heat conduction.

1. Introduction

The heat transfer in skin tissue is a very complex process. It involves multiple phenomenological mechanisms including conduction in tissue, convection between blood and tissues, blood perfusion or advection and diffusion through micro vascular beds, and metabolic heat generation [1]. The analysis of heat transfer and thermal damage in skin tissue are of great importance and can be useful for heat therapy in clinics.

In general, there are three kinds of bio-heat transfer models for skin tissues:(1) the Pennes model [2], (2) the C-V model [3] and (3) the DPL model [4]. Firstly, the heat conduction in biological tissue is modeled in the Pennes model which is based on the Fourier's law

$$q\left(\vec{r},t\right) = -k\nabla T\left(\vec{r},t\right) \tag{1}$$

The Pennes model was widely used in bio-heat transfer because of its simplicity. However, there are some disadvantages for the Pennes model as it assumes that the propagation speed of thermal disturbance is infinite. In fact, heat is always found to propagate with a finite speed within living biological tissues as they have highly non-homogeneous inner structure [5].

Secondly, using the concept of a finite heat propagation speed, Cattaneo [6] and Vernotte [7] simultaneously modified the Fourier law to form the C-V model:

$$q\left(\vec{r},t+\tau_q\right) = -k\nabla T\left(\vec{r},t\right) \tag{2}$$

Eq. (2) assumes that the heat flux and the temperature gradient occur at different times. The delay between the heat flux and temperature gradient is defined as the thermal relaxation time τ_q. Several literature are devoted to determine the phase lag for the heat flux [8–12]. The thermal wave theory ensures a strong path dependency for the temperature gradient. Although the C-V model can capture the microscale response in time, but not in space [13]. The C-V model is debatable in view of the fast-transient response with microstructural interaction effects [14].

Thirdly, in order to consider the effect of microstructural interactions in fast transient process of heat transport, Tzou [14]

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Nomenclature

c	specific heat of skin tissue,
c_b	specific heat of blood
f_i	dimensionless heating at the boundary, $LF_i(t)/kT_a$
h	heat convective coefficient
k	thermal conductivity of skin tissue
L	thickness of skin
N_{u0}, N_{uL}	Nusselt number,h_0L/k, h_LL/k
p	heat transfer per unit area of pulsed laser heating
q_{met}	metabolic heat generation in the skin tissue
$\overline{q}_{met}$	dimensionless metabolic heat generation in the skin tissue, $q_{met}L^2/kT_a$
q_{ext}	heat source due to external heating
$\overline{q}_{ext}$	dimensionless heat source due to external heating, $q_{ext}L^2/kT_a$
t	time variable
T_a,T	temperatures of arterial blood and skin tissue respectively
T_∞	temperatures of cooling fluid
u	step function
x	position coordinate
ξ	dimensionless position coordinate, x/L
ρ	density of skin tissue
ρ_b	density of blood
τ	dimensionless time,$kt/\rho cL^2$
τ_q	phase lag for heat flux, α/C_t^2
$\overline{\tau}_q$	dimensionless phase lag for heat flux, $k\tau_q/\rho cL^2$
τ_T	phase lag for temperature gradient
$\overline{\tau}_T$	dimensionless phase lag for temperature gradient, $k\tau_T/\rho cL^2$
ψ	ratio of the thermal conductivity to the heat capacity, $k/\rho cL^2$
Θ	dimensionless temperature of skin, $(T-T_a)/T_a$
Ω	index of thermal injury,
ϖ_b	the blood perfusion rate
$\overline{\omega}_b$	dimensionless blood perfusion rate, $L^2\varpi_b\rho_bc_b/k$

established a dual phase thermal lag (DPL) model that allows either the temperature gradient to precede heat flux vector or the heat flux vector precede temperature gradient. The model is expressed as

$$q\left(\overrightarrow{r},t+\tau_q\right)=-k\nabla T\left(\overrightarrow{r},t+\tau_T\right) \tag{3}$$

where τ_q is the phase lag for the heat flux vector, and τ_T is the phase lag for the temperature gradient. If the local heat flux vector results in the temperature gradient at the same location but an early time ($\tau_q > \tau_T$), the heat transfer is gradient-precedence type. On the other hand, if the temperature gradient results in the heat flux at a later time ($\tau_q < \tau_T$), the heat flow is called flux-precedence type. Antaki [4] and Xu et al. [14]have proposed dual-phase-lag model of bio-heat transfer (DPL) that allowed either the heat flux vector precede temperature gradient or the temperature gradient to precede heat flux vector. Afrin et al. [1] presented a generalized dual-phase lag model for living biological tissues based on non-equilibrium heat transfer between tissue, arterial and venous bloods. It was found that the phase lag time is greatly dependent to the properties of tissue and blood.

Due to its complexity, the approximated methods such as the meshless radial basis collocation method (RBCM), the finite element method (FEM), the finite difference method (FDM), the boundary element method (BEM) and the statistical method are often considered in solving the complex bio-heat transfer 3D problems. However, the analytical solution is helpful for precisely investigating the problem. The solution methods are reviewed as follows:

Xu et al. [15] and Ng and Chua [16] investigated the Pennes bio-heat transfer model by using the finite difference method and the finite element method. Deng and Liu [17] have obtained analytical solutions for Pennes bio-heat transfer equation by using Green's function method. Özen et al. [18] investigated the bio-heat transfer in the C-V model by using finite difference method. Ahmadikia*et al.* [19] presented the analytical solution of heat conduction on skin tissue in C-V model. Xu et al. [20] employed the finite difference method to solve DPL bio-heat transfer equation. Liu et al. [21] solved DPL bio-heat transfer equation by using method of Laplace transform. Lin [22] presented the analytical method for the heat conduction of skin subjected to harmonic heating in the dual-phase lag model. Kumar et al. [23] solved DPL bio-heat transfer equation by Finite element wavelet Galerkin method. Hooshmand et al. [24] solved the DPL bio-heat transfer with the isolated boundary condition by employing the separation of variables and Duhamel's integral method for both absorbing and scattering tissues. Kumar et al. [25] solved the Pennes, C-V and DPL models for the heat conduction of skin with three different constant boundary conditions by the Laplace transform method. The boundary condition is not time-dependent. The values of metabolic and spatial heat source in boundary condition of different thermal therapies were evaluated. Dombrovsky and Timchenko [26] investigated the problem of laser induced hyperthermia of superficial tumors. So far, no analytical method is presented for the DPL heat conduction of skin subjected to time-dependent pulsed laser heating along with general boundary conditions.

In this paper, the analytical solutions of the Pennes, C-V and DPL models for pulsed laser heating will be derived. The influence of boundary conditions and phase lags on the prediction of temperature and thermal damage will be investigated. The conditions of pulsed laser heating skin for the thermal therapy will also be investigated.

2. Dual-phase-lag model

Governing equation in the DPL model is [14,22]

$$\rho c\tau_q\frac{\partial^2T}{\partial t^2}=k\frac{\partial^2T(x,t)}{\partial x^2}+\tau_Tk\frac{\partial^3T(x,t)}{\partial t\partial x^2}-\varpi_b\rho_bc_b(T-T_a)-\left(\tau_q\varpi_b\rho_bc_b+\rho c\right)\frac{\partial T}{\partial t}+\left(q_{met}+q_{ext}+\tau_q\frac{\partial q_{met}}{\partial t}+\tau_q\frac{\partial q_{ext}}{\partial t}\right),\quad 0<x<L,\ t>0. \tag{4}$$

where ρ, c, k are the density, specific heat and thermal conductivity of skin tissue, respectively; ρ_b, c_b are the density and specific heat of blood, ϖ_b is the blood perfusion rate (volume blood per unit mass of tissue per unit time); T_a and $T$are the temperatures of arterial blood and skin tissue respectively; q_{met} is the metabolic heat generation in the skin tissue and q_{ext} is the heat source due to external heating.τ_q is the phase-lag in establishing the heat flux and associated conduction through a medium.τ_T is the phase-lag in establishing