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An investigation on coupled thermoelastic interactions in a thick plate due to axi-symmetric temperature distribution under an exact heat conduction with a delay

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A B S T R A C T

Present work is concerned with a recently proposed heat conduction model: an exact heat conduction model with a single delay term. The main purpose of this paper is to examine the effects of the single phase-lag parameter/delay term on wave propagation in an infinitely extended thick plate due to axisymmetric temperature distribution applied in the lower and upper surfaces of the plate. The problem is formulated by using the new heat conduction model in such a way that other existing models can be extracted as special cases. Potential function concept including the Laplace and Hankel Transform method is used to find the solution in the transformed domain. Inversion of Hankel Transform is done analytically to obtain the solution in Laplace transform domain. By using a suitable analytical approach, the complete analysis on the wave propagation and discontinuities of different wave fields are found out. Furthermore, a numerical method for the Laplace inversion is applied to obtain the distributions of different physical fields like, temperature, displacement and stresses in the middle plane of the plate. Analysis of the results obtained under new model along with a comparison of the respective results obtained in other models is presented in detailed way. The findings in the present work are believed to bring out some lights concerning the new heat conduction model that involves a single delay parameter.

1. Introduction

The topic “generalized thermoelasticity” has attracted several researchers during last few decades. The objective of this theory is to minimize the shortcomings which are inherent in the classical thermoelasticity theory [1]. Out of these shortcomings, infinite speed of thermoelastic disturbance, insufficient thermoelastic response of a solid body to short laser pulses, inferior representation of thermoelastic nature at low temperature etc. are the most important points to be addressed. The generalized theories are specially designated by finite speed of propagation of thermal disturbance. Firstly, we would like to recall the first two theories developed by Lord and Shulman [2] and Green and Lindsay [3]. In the theory developed by Green and Lindsay [3], two thermal relaxation time parameters are introduced, where as one relaxation time parameter is introduced in the theory proposed by Lord and Shulman [2]. Another model is introduced by Hetnarski & Ignaczak [4] and it is known as low temperature thermoelastic model in which the heat flux and free energy depend on temperature, strain tensor and the heat flux. Subsequently, this model is also explained by the system of non linear field equations.

Later on, Green and Naghdi [5–7] proposed three models which are subsequently known as thermoelastic model of type GN-I, GN-II and GN-III. Out of these three, the first two models are the special cases of model-III. In these theories, temperature gradient and thermal displacement gradient are taken to be among the constitutive variables. The linearized version of model-I is closely related to the classical thermoelastic model where as in model-II, there is no dissipation of thermal energy which is caused by no change in internal energy. This implies that the internal rate of production of thermal entropy is taken to be identically zero. However, this is not the case in the most general model, i.e., in GN-III model. The proposed heat conduction law under GN-III model is of the form

$$\vec{q}(p,t) = -\left[K\vec{\nabla}T(p,t) + K^*\vec{\nabla}\nu(p,t)\right]$$

where $K > 0$, $K^* > 0$ are the material parameters known as the

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thermal conductivity and conductivity rate, respectively. Here, $\dot{\nu} = T$ and ν is termed as thermal displacement while $\overrightarrow{q}$ is the heat flux vector.

By taking into account the micro structural effects in heat transport process, Tzou [8] proposed the two phase-lag model. Here two different phase-lags, one for the heat flux vector while other one for the temperature gradient vector, have been introduced. Thus the modified Fourier law of heat conduction is given as

$$\overrightarrow{q}(p, t+\tau_q) = -\left[K\overrightarrow{\nabla}T(p, t+\tau_T)\right], \quad \tau_q > 0, \tau_T > 0$$

Above relation explains that gradient of temperature at a point p in the body at time $t+\tau_T$ corresponds to the heat flux vector at that point at time $t+\tau_q$. The micro structural interactions during heat transport phenomenon is captured by these two parameters where the micro structural effects like phonon scattering causes the delay time τ_T and τ_q is the delay time caused by fast-transient effects of thermal inertia. Clearly, the above relation reduces to Fourier law in the case when $\tau_q = \tau_T = 0$. Now, if we assume $\tau_q > 0$, $\tau_T > 0$ and a second order approximation for ∇T and $\overrightarrow{q}$ are taken, above equation can be written as

$$q_i + \tau_q \dot{q}_i + \frac{\tau_q^2}{2}\ddot{q}_i = -K\left[T_{,i} + \tau_T \dot{T}_{,i} + \frac{\tau_T^2}{2}\ddot{T}_{,i}\right]$$

Subsequently, Quintanilla [9] explained the stability of this approximated two phase-lag model. He studied four types of problems in which the first one is to find the frame where third order of heat conduction is well posed and second one is how we can prove the exponential stability while third one is on spatial behavior of the solution of heat conduction in a semi infinite cylinder in R^3 and the last one is to determine a uniqueness of solution in the case of unbounded domain. Detailed review on the subject, generalized thermoelasticity and various non Fourier heat conduction models are reported in literature. Hetnarski and Ignaczak [10] reviewed thoroughly various models in a survey article by focusing on the theoretical significance of these models. They explained the domain of influence theorem with an initial boundary value problem for Lord-Shulman and Green-Lindsay theories. Chandrasekharariah [11,12] has also studied thoroughly this topic. In the review article by Chandrasekharariah [12], the dual phase-lag heat conduction model suggested by Tzou [8] has been extended to a hyperbolic thermoelastic model with dual phase-lag effects by using its Taylor series expansion and taking the second order term for $\overrightarrow{q}$ to ensure finite speed of thermal signal. Subsequently, in the frame of dual phase-lag thermoelastic model, an alternative extension of GN-III model is also introduced by Roychoudhuri [13]. This is called three phase-lag model in which three different phase-lag parameters are included in the constitutive equation of heat conduction proposed in GN-III model and this is of the form

$$\overrightarrow{q}(p, t+\tau_q) = -\left[K\overrightarrow{\nabla}T(p, t+\tau_T) + K^*\overrightarrow{\nabla}\nu(p, t+\tau_\nu)\right]$$

Here, the material parameter τ_ν is the additional delay time with respect to thermal displacement gradient $\nabla\nu$.

The above mentioned thermoelasticity theories have attracted the serious attention of researchers in recent years in order to find out several features of these models. Some qualitative analysis on these models are also reported. Quintanilla and Racke [14] discussed the stability of three phase-lag model of heat conduction equation and the effects of considering all these three material parameters. Dreher et al. [15] reported an analysis on dual phase-lag and three phase-lag heat conduction models and showed that when we combine the constitutive equations introduced in dual phase-lag and three phase-lag heat conduction theory with the energy equation, then there exists a sequence of eigenvalues in a point spectrum in such a way that its real parts tend to infinity [15]. This implies the ill-posed behavior of the problem in Hadamard sense and we can not find the continuous dependence results of the solution with respect to initial parameters. By mentioning these unacceptable results, Quintanilla [16] has recently proposed to reformulate the three phase-lag heat conduction model and suggested an alternative heat conduction theory with a single delay term. Quintanilla and Leseduarte [17] re-examined this new model given by Quintanilla [16] and found out the stability and spatial behavior of the solutions under this model. They considered $\tau_\nu < \tau_q = \tau_T$, and $\tau = \tau_q - \tau_\nu$, so that the constitutive law of heat conduction has been taken as

$$\overrightarrow{q}(t) = -[K\nabla T(t) + K^*\nabla\nu(t-\tau)]$$

By using the last equation, they have studied the spatial behavior of the solutions for this theory. A Phragmen-Lindelof type alternative [16] is found out and it is shown that the solutions either decay in an exponential way or blow up at infinity in an exponential way. The results are extended to a thermoelasticity theory by considering the Taylor series approximation of the equation of heat conduction to the delay term and Phragmen-Lindelof type alternative is obtained for both the forward and backward in time equations. Continuous dependence results for initial data and supply terms have been proved for this case. The continuous dependence results are further extended to the thermoelastic case.

Quintanilla [16] has further considered the Taylor series approximation until order l in the thermal gradient part of the constitutive law and reduced it in the form

$$\overrightarrow{q}(t) = -\left[K\nabla T(t) + K^*\left\{\nabla\nu(t) + \tau\nabla\dot{\nu}(t) + \ldots + \frac{\tau^l}{l(l-1)\ldots 1}\nabla\nu^{(l)}\right\}\right]$$

If this equation is adjoined with the energy equation, the new heat conduction equation is obtained as

$$c\dot{T}(t) = -\left[K\Delta T(t) + K^*\left\{\Delta\nu(t) + \tau\Delta T(t) + \ldots + \frac{\tau^l}{l(l-1)\ldots 1}\Delta T^{(l-1)}\right\}\right]$$

where c is the specific heat, Δ is the Laplacian operator. Quintanilla has also shown that the solutions of this heat conduction equation are always stable (at least) whenever $l \le 3$.

When we take $l = 0$, then this equation reduces to the form

$$c\dot{T}(t) = -[K\Delta T(t) + K^*\Delta\nu(t)]$$

This is the heat conduction equation under GN-III model.

When we take $l = 2$, we get the following equation of heat conduction which we refer to new model-I (i.e., Quintanilla model-I):

$$c\dot{T}(t) = -\left[K\Delta T(t) + K^*\left(1 + \tau\frac{\partial}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2}{\partial t^2}\right)\Delta\nu(t)\right]$$

If we neglect the term containing τ^2 for smallness, then we get the following equation which we refer to new model-II (i.e., Quintanilla model-II):