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A physics-based integer-linear battery modeling paradigm

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HIGHLIGHTS

- We describe a dispatch optimization paradigm for a hybrid energy system.
- We provide a detailed battery model.
- We simplify the battery model for inclusion in an optimization model.
- We measure the error between the simplified battery model and the detailed model.
- We show that our simplified model yields minimal error.

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ABSTRACT

Optimal steady-state dispatch of a stand-alone hybrid power system determines a fuel-minimizing distribution strategy while meeting a forecasted demand over six months to a year. Corresponding optimization models that integrate hybrid technologies such as batteries, diesel generators, and photovoltaics with system interoperability requirements are often large, nonconvex, nonlinear, mixed-integer programming problems that are difficult to solve even using the most state-of-the-art algorithms. The rate-capacity effect of a battery causes capacity to vary nonlinearly with discharge current; omitting this effect simplifies the model, but leads to over-estimation of discharge capabilities. We present a physics-based set of integer-linear constraints to model batteries in a hybrid system for a steady-state dispatch optimization problem that minimizes fuel use. Starting with a nonlinear set of constraints, we empirically derive linearizations and then compare them to a commonly used set of constraints that assumes a constant voltage and neglects rate-capacity. Numerical results demonstrate that assuming a fixed voltage and capacity may lead to over-estimating discharge quantities by up to 16% compared to our overestimations of less than 1%.

1. Introduction

Optimal steady-state dispatch of a stand-alone hybrid power system determines a fuel-minimizing power distribution strategy while meeting a forecasted demand over six months to a year. Corresponding optimization models that integrate hybrid technologies such as batteries, diesel generators, and photovoltaics (PV) with system interoperability requirements are often large, nonconvex, nonlinear, mixed-integer programming (MINLP) problems that are difficult to solve [5,29]. Mixed-integer programs (MIPs) that employ linearization and approximation techniques to bound and solve a simplified nonlinear problem frequently serve as tractable modeling alternatives to MINLPs. Although a MIP does not guarantee the feasibility and/or optimality of the solution to the corresponding nonlinear problem, it can still be an effective modeling option.

Batteries are integral to hybrid systems because of their ability to both store energy and provide on-demand, dispatchable power [10,21]. There are a variety of ways to effectively dispatch batteries in hybrid systems such as load-leveling, peak-shaving, and/or load-following [7], all of which store energy to minimize the use of additional technologies with start-up and ramping costs. Subject to a power loss of up to 10%, all batteries generate a direct-current (DC) power that requires conversion to alternating-current (AC) power to meet demand. Batteries store energy in electrochemical form in ways that differ by type; common battery chemistries associated with hybrid systems are nickel cadmium, nickel-metal hydride, lead acid, and lithium-ion. Each has its own documented advantages [35,39], e.g., cost, safety, performance, and size. We

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use data from lithium-ion and lead-acid battery manufacturers to represent baseline products within the industry.

Steady-state dispatch modeling for hybrid-power systems often assumes that the instantaneous fluctuations in frequency and voltage over time do not affect power dispatch. When a battery discharges, the electro-active species oxidizes at the anode and reduces at the cathode. The chemistry of the electrodes determines the voltage of an individual cell. Because concentrations of the electro-active species play a large role in determining the voltage and capability of a battery, detailed battery models often track concentrations in various parts of the battery. These diffusion-based models are accurate, because they take into account the transient voltage behavior of the battery, such as the voltage profile when switching between charge and discharge. The time for diffusion, i.e., the dissipation of the concentration gradient, is estimated by the quotient of the square of the battery electrolyte thickness and a coefficient that accounts for the diffusivity of the active species. The diffusivity of lithium ions in the electrolyte is on the order of 10^{-10} m^2/s, while the scale for different battery components is around 100 μm. This gives diffusion-based models a timescale, length^2/diffusivity, on the order of 100 s, which, when compared at the hourly time fidelity of our steady-state application, represents too short of a time interval for it to impact the accuracy of the model.

Although steady-state assumptions facilitate simplifying some complex relationships through linearization [8,13], modeling battery performance often requires both nonlinear and integer considerations. We recognize that lifetime is also an important characteristic in battery modeling; however, lifetime data is limited and difficult to properly test. The focus of this paper is on the set of constraints within a hybrid system optimization model that dictate battery performance, which are nonlinear. For more details, we refer the reader to [33]. Energy, which describes the total amount of electricity a source can provide, and power, which represents the total energy consumed per unit time, are functions of voltage. Although voltage varies with time, our steady-state assumptions render this variance negligible. However, voltage also varies depending on state of charge (SoC), i.e., the fraction of battery capacity available for discharge, and current; failing to consider this variability causes an over-estimation of battery resource availability. By employing a physics-based definition, in which average power is the product of current and voltage, and Ohm's law, which relates current and resistance to voltage, we can model the rate and associated amount of electricity available from batteries in terms of a single independent variable, i.e., current.

A battery's capacity varies based on the magnitude of the discharge current. The higher the current draw, the less total capacity available, which we refer to as the rate-capacity effect. Peukert's equation [12], which accounts for the rate-capacity effect in lead-acid batteries by exponentially relating battery capacity to discharge current, could also be applied to other battery chemistries. The Kinetic Energy Battery Model (KiEBM), which portrays the change in capacity as a nonlinear function of charge and discharge rates, models a battery's capacity as two tanks, one of which is immediately available for discharge and the other of which is chemically bound [22]. Alternately, the CIEMAT (Centre for Energy, Environment and Technology in Madrid, Spain) model presents a nonlinear set of equations that accounts for dynamic and complex battery operations [9]. The common theme among these battery representations is that neglecting rate-capacity leads to over-estimating the performance of the battery, which may yield an infeasible dispatch solution, i.e., battery discharge power that is not actually available. Researchers tend to use constraint sets in a hybrid model that neglect rate-capacity (see Section 2 for cases in which tractability is a concern). These large-scale MIPs minimize costs in design and/or dispatch problems that determine optimal technology procurement and/or operation to satisfy demand [6,16–20,25,27,30,31,34,37,38]; they may also approximate unit commitment that schedules technologies to meet demand [15,23,36].

Recognizing the pitfalls of over-simplifying battery performance and the potential impact rate-capacity error has on the solution, we first introduce simple *energy* constraint set ($\mathcal{E}$), which not only neglects rate-capacity effects, but assumes a constant voltage. As an alternative to ($\mathcal{E}$), we present a *nonlinear* set of constraints ($\mathcal{N}$). Next, we linearize the nonlinear, nonconvex relationships in ($\mathcal{N}$) to empirically derive a *physics-based* set of constraints ($\mathcal{P}^{\mathcal{B}}$). We then determine the theoretical error associated with both ($\mathcal{E}$) and ($\mathcal{P}^{\mathcal{B}}$). Lastly, we present ($\mathcal{P}^{\mathcal{F}}$) (see Appendix), which includes a *fuel*-minimizing objective function and a set of mixed-integer constraints for system interoperability and PV and generator technologies. The combination of ($\mathcal{P}^{\mathcal{B}}$) with ($\mathcal{P}^{\mathcal{F}}$) and ($\mathcal{E}$) with ($\mathcal{P}^{\mathcal{F}}$) forms two optimization models, which we term ($\mathcal{P}^{\mathcal{B}^+}$) and ($\mathcal{E}^+$), respectively, for the hybrid power steady-state dispatch problem. We solve 12 scenarios and compare battery dispatch solutions of each model by quantifying the error in each.

The remainder of this paper is organized as follows: Section 2 presents ($\mathcal{E}$); Section 3 reveals our physics-based mixed-integer, nonlinear battery constraints ($\mathcal{N}$) presented in MINLP format. Section 4 details how we derive ($\mathcal{P}^{\mathcal{B}}$) from ($\mathcal{N}$). Section 5 presents scenarios and results, while Section 6 concludes.

2. A commonly used set of battery constraints ($\mathcal{E}$)

In this section, we present a commonly employed set of mixed-integer battery constraints ($\mathcal{E}$) in which voltage is constant and SoC serves as an unrestricted proxy for the fraction of capacity available for discharge. We use lower-case letters for parameters and upper-case letters for variables. We also use lower-case letters for indices and upper-case script letters for sets. Superscripts and accents distinguish between parameters and variables that utilize the same base letter, while subscripts identify elements of a set. The units of each parameter and variable are provided in brackets after its definition, where applicable.

Sets

$t \in \mathcal{T}$	a single time period within the set of time periods

Parameters

τ	length of one time period [h]
$\bar{e}$	manufacturer energy maximum rated-capacity of the battery [W h]
η^+, η^-	conversion efficiency of power flow into and out of the battery, respectively

Variables

B_t^{soc}	SoC of the battery in time period t
P_t^+, P_t^-	aggregate power into and out of the battery in time period t, respectively [W]
B_t^+, B_t^-	1 if the battery is charging or discharging, respectively, in time period t, 0 otherwise

Constraints ($\mathcal{E}$)

$$\bar{e}B_t^{soc} = \bar{e}B_{t-1}^{soc} + \tau(\eta^+ P_t^+ - P_t^-) \quad \forall t \in \mathcal{T} : t > 1 \tag{1a}$$

$$\tau P_t^- \leqslant \bar{e}B_{t-1}^{soc} \quad \forall t \in \mathcal{T} : t > 1 \tag{1b}$$

$$\tau P_t^+ \leqslant \bar{e}(1 - B_{t-1}^{soc}) \quad \forall t \in \mathcal{T} : t > 1 \tag{1c}$$

$$\tau P_t^- \leqslant \bar{e}B_t^- \quad \forall t \in \mathcal{T} \tag{1d}$$

$$\tau P_t^+ \leqslant \bar{e}B_t^+ \quad \forall t \in \mathcal{T} \tag{1e}$$

$$B_t^{soc} \leqslant 1 \quad \forall t \in \mathcal{T} \tag{1f}$$