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A single domain formulation on conjugate heat transfer in parallel plate microchannel with electrical double layer

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In this study, the conjugate conduction-convection heat transfer in the thermally developing region of a parallel plate microchannel with the constant heat flux at the walls is investigated. The flow is forced to move by applying the pressure difference and is assumed to be fully developed. Moreover, the effects of electrical double layer (EDL) with low zeta potential on the fluid flow and on the temperature field are taken into account. To do so, an analytical/numerical method is proposed to solve the energy equation in a single domain formulation. The energy equation is split and a series solution based on the variational calculus is employed to determine the temperature field. The effects of various parameters such as Peclet number, non-dimensional wall thickness, thermal conductivity ratio, and the zeta potential are studied in details. The obtained results show that the increasing of zeta potential decreases the heat transfer rate. Moreover, the Nusselt number has a non-monotonic behavior when the wall thickness varies.

1. Introduction

The fluid flow and heat transfer in micro-channels have various applications in electronics, bio-engineering, advanced fuel cells, and micro heat exchangers. Therefore, it has been investigated by researchers recently. In micro-scale heat transfer, the effect of axial conduction is not negligible because the wall thickness is comparable to the hydraulic diameter of the channel. So, the conduction heat transfer in the wall is not negligible in comparison with the convective heat transfer, and the mechanism of heat transfer becomes conjugate. Also, the effects of conduction heat transfer are increased when the Peclet number (*Pe*) is not too large which is occurred in microchannels. Moreover, in the microchannels, the electro-viscous effects which stem from the presence of electrical double layer (EDL) and its interaction with the ions in the fluid should be considered.

Conjugate heat transfer in microchannels has been investigated in many studies. Maranzana et al. [1] investigated the heat transfer between parallel plates with considering the conduction in the walls. They proposed two analytical models to analyze this type of problem. In another study [2], they showed that the conduction becomes more important when the thickness of walls is increased. Extended Graetz problem including wall conduction in a parallel plate channel was analytically investigated by Weigand and Gassner [3]. Cole and Cetin [4] studied the conjugate heat transfer between parallel plates with the uniform heat flux boundary condition using the method of Green's functions. Temperature field in a parallel-plate microchannel with steady-periodic heating at the wall was expressed analytically in the form of integrals; also numerical results are obtained by quadrature to explore the effects of heating frequencies, and wall properties in the study of Cole [5]. Seddiq et al. [6] used a Lattice-Boltzmann method to analyze the conjugate heat transfer in a channel with infinite number of heated obstacles mounted on the wall.

The three-dimensional conjugate heat transfer in a rectangular microchannel was studied numerically in the study of Li et al. [7]. They used a finite difference method with a Tri-Diagonal Matrix Algorithm (TDMA) to investigate the thermal entrance length and the effects of thermophysical properties of fluid on heat transfer. The conjugate heat transfer in the rectangular microchannel was also studied by Kosar [8]. He showed that the thermal conductivity in comparison to the substrate thickness has a greater influence on conduction. The conjugate heat transfer in microtubes has been investigated in some studies [9,10]. Rahimi and Mehryar [11] numerically studied the effects of thermal conductivity and wall thickness on the Nusselt numbers at the entrance and ending regions of a circular cross-section microchannel with the constant

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Nomenclature

A	area of the microchannel cross section (m^2)
A_1	area of the fluid part of the microchannel cross section (m^2)
A_2	area of the solid part of the microchannel cross section (m^2)
C_p	specific heat capacity (J/kgK)
E_p	electrokinetic potential (V)
E_x	electrokinetic potential gradient in the x direction (V/m)
e	charge of a proton (C)
F_n	eigenfunction
f	basis function
H	microchannel height (m)
I_c	conduction current (A/m)
I_{st}	streaming current (A/m)
K	electrokinetic separation distance
k	Debye-Huckel parameter (1/m)
k_b	Boltzmann constant (J/K)
L	microchannel length (m)
$\mathfrak{n}$	non-dimensional form of the outward normal direction to the wall
n_∞	bulk ionic number concentration ($1/m^3$)
Nu	Nusselt number
p	pressure (N/m^2)
Pe	Peclet number
q	heat flux at the wall of microchannel (W/m^2)
T	temperature (K)
T_{in}	inlet temperature (K)
T_{rf}	reference temperature, room temperature (K)
U	mean axial velocity (m/s)
u	axial velocity (m/s)
W	thickness of the microchannel (m)
x,y	coordinates (m)
z	ionic valence
α	a dimensionless parameter in the dimensionless expression of temperature
β_n	eigenvalue
Γ	dimensionless microchannel boundary
δ	dimensionless wall thickness
ε	fluid permittivity (C/Vm)
ε_0	permittivity of vacuum (C/Vm)
ε_r	dielectric constant of the fluid
ζ	zeta potential (V)
θ	dimensionless temperature
θ_1, θ_2	first and second parts of the dimensionless expression of temperature, Eq. 31
θ_3, θ_4	first and second parts of the dimensionless expression of temperature, Eq. 46
θ_m	dimensionless bulk temperature
θ_W	dimensionless temperature of wall inner surface
λ	thermal conductivity (W/mK)
λ_0	thermal conductivity ratio
λ_b	fluid electrical conductivity ($1/\Omega m$)
λ_D	characteristic thickness of the EDL (m)
λ_σ	surface conductivity ($1/\Omega$)
μ	dynamic viscosity (kg/ms)
ρ	fluid density (kg/m^3)
ρ_e	net volume charge density (C/m^3)
σ	dimensionless thickness of the microchannel
ψ	electrical field potential (V)

Subscripts

f	fluid part of the microchannel
s	solid part of the microchannel

Superscript

*	dimensionless variable

heat flow rate at the outer surface of wall. They showed that axial wall conduction decreases the Nusselt numbers at the entrance and ending regions. Kroeker et al. [12] compared the conjugate heat transfer in rectangular and circular microchannels. The temperature field in both fluid and solid regions of trapezoidal microchannel with surface roughness was numerically and experimentally investigated in the study of Qu et al. [13].

The effects of viscous heating on conjugate heat transfer and the averaged Nusselt number in microchannels were studied by Morini [14]. He presented a model to predict the temperature rise along a microchannel due to viscous heating. In addition, Avcı et al. [15] used the finite volume method to investigate the effects of viscous dissipation on temperature field and interface heat flux in microtubes. They showed that the increasing of viscous dissipation decreases the Nusselt number. The conjugate heat transfer by considering the viscous heating in slit microchannel with the upper plate movement was also analyzed by the minimization of the entropy generation in the study of Mondal and Dholey [16].

The conjugate heat transfer with jump conditions in the microchannel was solved using separation of variables by Veeraragavan and Cadou [17]. They investigated the effects of flow velocity on the temperature field. In addition, the effects of slip flow and wall heat flux boundary condition were analytically investigated in the study of Ibáñez et al. [18]. The conjugate heat transfer in parallel plate microchannel with a constant heat flux boundary condition considering slip velocity and temperature jump boundary condition with different Knudsen numbers was numerically investigated by Kabar et al. [19]. Bushehri et al. [20] have investigated the effects of temperature jump boundary condition on the conjugate heat transfer in parallel plate microchannel with the constant heat flux boundary condition by proposition of a solution method for the coupling equations and using the finite volume method. The conjugate heat transfer in microtube subjected to an axially varying heat flux considering the slip velocity and the temperature jump boundary condition was solved using the finite volume method in the study of Aydin and Avci [21]. Also, they considered the effects of viscous dissipation in their analysis. They showed that the increasing of Knudsen number decreases the Nusselt number due to rarefaction.

Effects of electrical double layer on the conjugate heat transfer in microchannels under conditions of pressure-driven flow are not investigated in the above studies. In the present study, the conjugate heat transfer in the thermally developing region of a parallel plate microchannel with the constant heat flux at walls as shown in Fig. 1 is investigated. Moreover, the effects of electrical double layer (EDL) with low zeta potential are taken into account. The electrical double layer affects on the fluid flow. Modification of flow field by considering the presence of electrical double layer will influence on the temperature field and consequently on the heat transfer. Since the presence of electrical double layer affects on the fluid flow and heat transfer in microchannels, considering these effects offers more accurate values for the heat transfer rate. The fluid is aqueous