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The extended Graetz problem for micro-slit geometries; analytical coupling of rarefaction, axial conduction and viscous dissipation

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A B S T R A C T

In order to support the recent MEMS and Lab-on-a-chip technologies, we studied heat transport in micro-scale slit channel gas flows. Since the micro convection transport phenomena diverges from conventional macro-scale transport due to rarefaction, axial conduction and viscous heating, an accurate understanding requires a complete coupling of these effects. For such cases, we studied heat transfer in hydrodynamically developed, thermally developing gas flows in micro-slits at various flow conditions. The analytical solution of the energy equation considered both the heat conduction in the axial direction and heat dissipation of viscous forces. Furthermore, updated boundary conditions of velocity slip and temperature jump were applied based on Knudsen number of flow in order to account for the non-equilibrium gas dynamics. Local Nusselt number (*Nu*) values were calculated as a function of Peclet (*Pe*), Knudsen (*Kn*) and Brinkman (*Br*) numbers which were selected carefully according to possible micro-flow cases. Strong variation of *Nu* in thermal development length was found to dominate heat transfer behavior of micro-slits with short heating lengths for early slip flow regime. For this instance, influence of axial conduction and viscous dissipation was equally important. On the other hand, high *Kn* slip flow suppressed the axial conduction while viscous heating in a small surface-gas temperature difference case mostly determined the fully developed *Nu* and average heat transfer behavior as a function of *Kn* value.

1. Introduction

The most recently developed small devices using micro-scale physics provide multiple advantages over their macro scale counterparts such as, lower capital costs and energy requirements, while offering higher efficiencies and simplicity in operations. However, as the size decreases, complications arise in ongoing transport behaviors. For example, the process of cooling micro electro mechanical systems (MEMS) or mixing/diffusion in Lab-On-a-Chip devices becomes more complex since the micro-level mass and heat transport displays very different behavior than conventional mechanisms. Specifically, micro gas flows diverge from Navier-Stokes solution due to the rarefaction effects. Additionally, low flow speeds and short heating lengths observed in micro flows further create axial conduction through thermal developing regions, while the large length to diameter ratios and small surface-gas temperature differences build viscous dissipation; such effects are mostly neglected for macro-scales. As a result, a complete coupling of the above listed micro-scale related effects is required to understand and characterize convective gas transport of numerous existing applications and to further design new ones.

Based on current micro fabrication techniques, the manufacturing of channels is simpler than microtubes. Controlling one dimension in micro levels is the task for most cases which makes micro-slits a common geometry for multiple applications; such as microbial fuel cells [1], methanol steam reforming and hydro-energy applications [2], and evaporators for micro-power generation systems [3]. In slit geometry, characteristic flow dimensions are represented by the hydraulic diameter defined as $D_H = (4A)/W$, where A is the cross-sectional area and W is the wetted perimeter.

Gas flows in micro-geometries with small D_H values undergoes non-continuum behaviors. Due to the surface-gas molecular collisions, non-equilibrium gas dynamics develops in near wall region known as Boltzmann Layer (BL). The BL extends from the surface on the order of gas mean free path (λ ~ 60 nm for air at standard

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conditions) which is a negligible region for most conventional macro cases. As the channel dimensions gets small, BL coverage in flow domain becomes non-negligible. Simply, the ratio of BL to characteristic flow dimensions can be considered as the level of non-equilibrium gas dynamics. In Kinetic Theory, Knudsen number ($Kn = \lambda/D_H$) functions in such a way by calculating the ratio of the mean free path to the characteristic dimension of the flow to characterize gas rarefaction. The flow is considered as a continuum flow for small values of Kn (<0.01) and the well-known Navier-Stokes equations are applicable for the flow field. However, the decrease of characteristic length of the system increases the Kn which results in the continuum approach to fail. For $0.01 < Kn < 0.1$, the flow is in slip-flow regime (slightly rarefied) where the fluid particles adjacent to the boundary surface are not in thermodynamic equilibrium with the wall, there would be slip velocity and temperature jump at the channel wall [4].

Multiple researchers implemented slip flow models successfully to consider the rarefaction effects on microscale flows [5]. Specifically, Bayazitoglu et al. [6], Rosa et al. [7], Cetin et al. [8], Cetin [9], Haddout and Lahjomri [10] and Yu and Ameel [11] observed that velocity slip increases the Nusselt number ($Nu = (h \times D_H)/k$, where h is convective and k is the conductive heat transfer coefficients of fluid), while the temperature jump has a negative effect on heat transfer in single phase microchannel flows.

Rarefaction is not the only complication developed in micro-confinements. There are additional mechanisms which become dominant at micro-scales. The first one is the heat transfer occurring through the thermal development length, which is a major mechanism in many micro-scale heat exchangers. Originally known as the Graetz Problem, the thermal entrance region of a tube flow at a constant surface temperature was first investigated by Graetz [12] and Nusselt [13]. The authors both worked on hydrodynamically developed and thermally developing flow for a negligible axial conduction and viscous dissipation case. Since then, researchers have focused on to extension of the Graetz Problem to include additional heat transfer mechanisms, specifically for micro-scale flows. The second of the major micro flow effects is axial conduction. This effect especially dominates thermally developing region of heat transfer. The small Reynold numbers ($Re = (V \times D_H)/\nu$, where V is flow speed and ν is kinematic viscosity) of the micro-flows results in small Peclet numbers ($Pe = Re \times Pr$ where Pr is the Prandtl number); and hence, the convection term no longer dominates the conduction term in the axial direction. Analytical consideration of the Graetz Problem with axial conduction is challenging due to the resulting non-self-adjoint eigenvalues [14]. Third, the small confinements create large length to diameter ratios which develops viscous heating, and the effect of viscous dissipation becomes significant as the wall-to-fluid temperature difference becomes small in micro-convection (non-negligible Brinkman number). The viscous dissipation prolongs the thermal development through the flow direction and dominates the fully develop heat transfer.

Multiple studies exist concluding the effects of rarefaction combined with thermal development region, axial conduction and viscous dissipation for microscale flows. By adding one or two or three of the aforementioned effects, the so called extended Graetz problems were developed. Specifically, rarefaction and axial conduction for fully developed flows in microchannels studied by Hadjiconstantinou and Simek [15] where fully developed Nu values found to be increasing by axial conduction. Rarefaction and axial conduction were also studied for thermally developing region by Huang et al. [16], Kakac et al. [17] and Cole et al. [18] where axial conduction was found to increase the temperature at the entrance of the channel and enhance the heat transfer. Jeong and Jeong [19], Koo and Kleinstreuer [20] and Chen [21] studied viscous effects with rarefaction and reported that fully developed Nusselt number increases under viscous dissipation in the presence of small Knudsen numbers (slip flow $0.01 < Kn < 0.1$). Similarly, Cetin et al. [8,22] focused to extend the Graetz problem by including all three micro-scale effects. With a similar perspective, Haddout and Lahjomri [10] studied rarefaction, axial conduction and viscous dissipation effect on the Graetz problem in tube and channel geometries mostly for a case of finite heating region which is comparable with the corresponding channel height. However, there is a knowledge gap for a proper coupling of all micro-scale effects for a micro-slit convective flow. Especially, heat transfer in micro-slits at low Pe with non-negligible viscous heating through thermal development length which occasionally extends up to ten times the flow height needs to be calculated. Recently, we solved the mathematical challenge and performed a complete analytical coupling of these rarefaction, axial conduction and viscous dissipation effects for the Graetz problem in micro-tubes [23].

The objective of this study is to further derive earlier found analytical approach to devise a coupled solution for the rarefaction, viscous dissipation, and axial conduction effects on a hydrodynamically developed thermally developing micro-slit gas flow. An analytical solution will be obtained by eliminating existing mathematical challenge by using Gram–Schmidt orthogonalization accompanied with the Gauss quadrature method. Convective micro-slit gas flow in the slip-flow regime will be solved with constant wall temperature. The confluent hypergeometric functions will be employed to solve the energy equation in order to provide a fundamental understanding of the effects of the non-dimensional parameters on heat transfer characteristics. Extension of thermally developing region in the axial direction is carefully analyzed at the presence of all microflow effects with various flow conditions.

2. Analysis

The studied geometry of the slit microchannel of hydrodynamically fully developed and thermally developing gas flow can be found in Fig. 1. The fully developed velocity profile develops through the unheated part.

For the slip flow regime, slip-velocity and temperature-jump boundary conditions were defined by Maxwell [4] as can be seen in Eqs. (1) and (2),

$$u_s = -\frac{2-\sigma_m}{\sigma_m}\lambda\left(\frac{du}{dy}\right)_{y@surface} \tag{1}$$

$$T - T_s = -\frac{2-\sigma_t}{\sigma_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\left(\frac{\partial T}{\partial y}\right)_{y@surface} \tag{2}$$

By using the slip and temperature jump boundary conditions, the non-dimensional fully developed velocity profile is obtained as follows:

$$u^* = \frac{u}{u_{mean}} = \frac{3}{2}\frac{\left(1 + 8Kn - \left(\frac{y}{H}\right)^2\right)}{1+12Kn} \tag{3}$$

Energy equation with the effects of viscous dissipation and axial conduction, and the boundary conditions for the flow system can be written as;

$$u\frac{\partial T}{\partial x} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\gamma}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{4a}$$